Outline

- Review of small signal modeling
- Large signal considerations
- Application of $V_{dsat}$ and $\Delta V$ analysis to CMOS subcircuits
A Common Source Amplifier

Review

- What is the purpose of including $M_2$ and $M_3$?
- How do we calculate the gain of this amplifier?
Small Signal Analysis

- Plug in Thevenin equivalents of the different transistors
  - Why is the Thevenin model for $M_2$ absent in the above figure?
- Gain is readily calculated as

$$v_{out} = -g_{m1}(r_{o1}||r_{o3})v_{in}$$
Resulting Small Signal Amplifier Model

According to our small-signal analysis

- $v_{out}$ equals zero when $v_{in}$ equals zero
- We can vary $v_{in}$ across an unlimited range and $v_{out}$ is proportionally amplified

Is this reality?
In reality
- $V_{out}$ is not zero for $V_{in}$ equal to zero
- The range of $V_{out}$ is bounded by the supply voltage
- The relationship between $V_{in}$ and $V_{out}$ is nonlinear
  - $M_1$ and $M_3$ enter into different regions of operation depending on the value of the output voltage

Key assumption in our small signal model
- Devices are in saturation


**Examination of ΔV and V_{dsat} for Large Signal Analysis**

\[
\Delta V = V_{GS} - V_T
\]

Overdrive Voltage: \(\Delta V = V_{GS} - V_T\)

- Strong inversion: \(\Delta V = \sqrt{\frac{2I_D L}{\mu_n C_{ox} W}}\)
- Weak inversion: \(\Delta V < 0\)

Saturation Voltage: \(V_{dsat}\)

- Strong inversion: \(V_{dsat} = \Delta V\)
- Weak inversion: \(V_{dsat} \approx 100 \text{ mV}\)
Define the linear range of the amplifier as the region within which both $M_1$ and $M_3$ are in the saturation region.
- Use $V_{dsat}$ to define this range
- Note that this is an approximation for hand calculations
  - Curve is actually nonlinear even within this range

The key parameters in range calculations will be $V_{dsat}$ and $\Delta V$.
Example 2: Source Follower Circuit

- Small signal analysis follows from Thevenin modeling
- Over what input and output voltage range do we assume that the above model is reasonably accurate?
Calculation of Input and Output Range

- **Input range**
  - To keep $M_1$ and $M_2$ in saturation
    \[ V_{in} > V_T + \Delta V_1 + V_{dsat2} \]
  - Supply constraint
    \[ V_{in} < V_{dd} \]

- **Output range**
  \[ V_{dsat2} < V_{out} < V_{dd} - (V_T + \Delta V_1) \]
Example 3: Common Gate Amplifier

- Small signal analysis follows from Thevenin model
  - Note: what is the relationship between $i_{in}$ and $V_{in}$?
- Key questions for large signal analysis
  - What should $V_{bias}$ be set to in order to maximize the output swing?
  - What is the resulting output swing?
Constraints on $V_{bias}$ and Output Range

- To keep $M_2$ and $M_4$ in saturation
  \[ V_{bias} - (V_T + \Delta V_1) > \max(V_{dsat2}, V_{dsat4}) \]
  \[ \Rightarrow V_{bias} > V_T + \Delta V_1 + \max(V_{dsat2}, V_{dsat4}) \]

- To keep $M_1$ in saturation
  \[ V_{out} - (V_{bias} - (V_T + \Delta V_1)) > V_{dsat1} \]
  \[ \Rightarrow V_{out} > V_{bias} - (V_T + \Delta V_1) + V_{dsat1} \]
Calculation of Maximum Output Range

- **Minimum** $V_{\text{bias}}$ allows the maximum output range

  $\Rightarrow V_{\text{bias}} = V_T + \Delta V_1 + \max(V_{\text{dsat}_2}, V_{\text{dsat}_4})$

- **Resulting output range**

  $V_{\text{bias}} - V_T < V_{\text{out}} < V_{\text{dd}}$

  $\Rightarrow V_{\text{dsat}_1} + \max(V_{\text{dsat}_2}, V_{\text{dsat}_4}) < V_{\text{out}} < V_{\text{dd}}$
Input Voltage Range of a Differential Amplifier

- Assume common-mode operation (i.e., $V_{in+} = V_{in-}$)
  - $M_4$ should remain in saturation
    \[ \Rightarrow V_{in+} > V_T + \Delta V_1 + V_{dsat4} \]
  - $M_1$ (and $M_2$) should remain in saturation
    \[ \Rightarrow V_{in+} < V_{dd} - I_{d1}R_1 + V_T + \Delta V_1 - V_{dsat1} \]
- How about differential-mode operation?
Small Signal Analysis of Differential Amplifier

- Key relationship is that between output current and input voltage
- Using the half-circuit technique

\[-i_{d1} = g_{m1} \frac{v_{id}}{2} \text{ (for } R_L \ll r_{o1}) \quad i_{d2} = -g_{m2} \frac{v_{id}}{2} \text{ (for } R_L \ll r_{o2})\]

- What is the large signal behavior for differential operation?
**Large Signal Behavior of Differential-Mode Operation**

- **Note:** above analysis assumes strong inversion
  - Problem Set 2 will consider weak inversion
Large Signal Analysis of Current Mirrors

- **Note**: above analysis assumes strong inversion
- Is accurate for weak inversion as well

\[
\frac{I_2}{I_1} = \frac{1}{2} \mu_n C_{ox} \frac{W_2}{L_2} \left( V_{GS2} - V_{T2} \right)^2 (1 + \lambda_2 V_{ds2}) + \frac{\Delta V_2}{\Delta V_1} \left( V_{GS2} - V_{T2} \right)^2 (1 + \lambda_2 V_{ds2})
\]

\[
\frac{I_2}{I_1} = \frac{1}{2} \mu_n C_{ox} \frac{W_1}{L_1} \left( V_{GS1} - V_{T1} \right)^2 (1 + \lambda_1 V_{ds1}) + \frac{\Delta V_1}{\Delta V_2} \left( V_{GS1} - V_{T1} \right)^2 (1 + \lambda_1 V_{ds1})
\]

But, \( V_{T} + \Delta V_1 = V_{T} + \Delta V_2 \) \( \Rightarrow \Delta V_1 = \Delta V_2 \)

\[
\frac{I_2}{I_1} = \frac{W_2}{W_1} \frac{L_1}{L_2} \left( 1 + \lambda_2 V_{ds2} \right) \left( 1 + \lambda_1 V_{ds1} \right)
\]

**Mismatch due to** \( V_{ds} \)**difference**

**Current setting based on geometry**

**Note**: for accurate ratio, set \( L_1 = L_2 \)
Small Signal Analysis of Current Mirrors

- **Relationship between** $i_1$ **and** $i_2$ (ignoring $r_{o2}$)

\[
i_2 = \frac{g_{m2}}{g_{m1}} i_1 \approx \frac{\sqrt{2\mu n C_{ox}(W_2/L_2)} I_2}{\sqrt{2\mu n C_{ox}(W_1/L_1)} I_1} i_1 \quad \text{or} \quad \frac{qI_2/(nkT)}{qI_1/(nkT)} i_1
\]

- **Strong Inv.**

- **Assuming** $L_1 = L_2$

\[
i_2 \approx \frac{W_2}{W_1} i_1
\]

- **Weak Inv.**

- **How is this analysis useful?**

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Consider A Simple Current Source

- Current is set by four variables (assume $L_1 = L_2$)
  - $V_{dd}$, $V_{gs1}$, $R_{bias}$, $W_2/W_1$
- Bias calculations

\[
I_1 = \frac{V_{dd} - V_{gs1}}{R_{bias}} = \frac{V_{dd} - (V_T + \Delta V_1)}{R_{bias}}
\]

- But $\Delta V_1$ depends on $I_1$, so must solve this iteratively
- Issue – it’s desirable to have current be independent of supply voltage
Impact of Supply Variations on Simple Current Source

- Analyze using small signal model

\[ i_2 = \frac{g_{m2}}{g_{m1}} i_1 \approx \frac{W_2}{W_1} i_1 = \frac{W_2}{W_1} R_{bias} + \frac{v_{dd}}{1/g_{m1}} \]

- The simple current source is extremely sensitive to supply variations!
Suppose We Made $I_1$ a Current Source?

- Much less sensitive to supply voltage variations
  - $R_{bias} = r_o$ is now quite large
- So, for robust biasing
  - The trick is to make a nice current source and then ship it to other circuits using current mirrors
  - We will discuss this issue in more detail in the context of bandgap references
The Issue of $V_{ds}$ Mismatch in Current Mirrors

- **Issue:** Current $I_2$ can vary significantly as a function of the drain voltage of $M_2$
  - We often want a tightly controlled current set only by $I_1$ and transistor sizes
- **How do we improve the current mirror matching performance?**

$$\frac{I_2}{I_1} = \frac{W_2}{W_1} \frac{(1 + \lambda_2 V_{ds2})}{(1 + \lambda_1 V_{ds1})}$$

Mismatch due to $V_{ds}$ difference

Current setting based on geometry

Note: we are assuming $L_1 = L_2$
Cascoded Current Sources

- Key transistor for determining \( I_2 \) is \( M_1 \)
  - Why is \( M_2 \) less important?
- Cascoding allows much closer match between \( V_{ds1} \) and \( V_{ds4} \) as drain voltage of \( M_2 \) is varied
  - Current \( I_2 \) is much better controlled
How does the above model allow you to infer that $I_2$ is more sensitive to gate variations impacting $M_1$ than those impacting $M_2$?
The Drawback of Cascoding

- Output voltage range is reduced
- Can we improve the voltage range?
**Improved Swing Cascode**

- **Key idea:** set size of $M_3$ such that $V_{ds1} = V_{dsat1}$
  - Assuming strong inversion for $M_1$ and $M_3$
    \[
    \Delta V = \sqrt{\frac{2I_dL}{\mu nC_{ox}W}} \Rightarrow \alpha = \frac{1}{4}
    \]
  - Assuming weak inversion for $M_1$ and $M_3$
    \[
    V_{dsat1} \approx 100\text{mV}, \quad \frac{\Delta V_{gs}}{I_{dens}} \propto 100\text{mV/dec} \Rightarrow \alpha = \frac{1}{10}
    \]
Alternative Implementation of Improved Swing Cascode

- Set $\alpha$ as on previous slide
- Note: both implementations share a common problem
The Issue of Current Mismatch

- The improved swing approach causes a systematic mismatch between $I_2$ and $I_1$
  - Key issue: $V_{ds1} \neq V_{ds4}$

- Can we fix this problem?

Recall:

$$\frac{I_2}{I_1} = \frac{W_2}{W_1} \frac{(1 + \lambda_2 V_{ds2})}{(1 + \lambda_1 V_{ds1})}$$

Mismatch due to $V_{ds}$ difference
Techniques to Reduce Current Mismatch

- Systematic mismatch between \( I_1 \) and \( I_2 \) is greatly reduced by using the above circuit (now \( V_{ds1} \approx V_{ds4} \))

- Additional techniques to reduce random mismatch between \( I_1 \) and \( I_2 \)
  - Set \( L_1 = L_4 >> L_{\text{min}} \)
    - Note: set \( L_2 = L_3 \approx L_{\text{min}} \) to lower area and capacitance
  - Set \( W_2/W_3 = I_2/I_1 \) so that \( \Delta V_2 = \Delta V_3 \)
Conclusion

- Analog circuit design involves both small signal and large signal modeling
  - Small signal modeling is limited by large signal considerations with respect to its accuracy
    - Hybrid-$\pi$ model (basis of our Thevenin models) assumes that the respective MOS device is in saturation
  - Large signal modeling is simplified by utilizing $V_{dsat}$ and $\Delta V$ analysis
    - Used to determine voltage ranges over which devices remain in saturation
    - Important in exploring issues of systematic mismatch in current mirrors