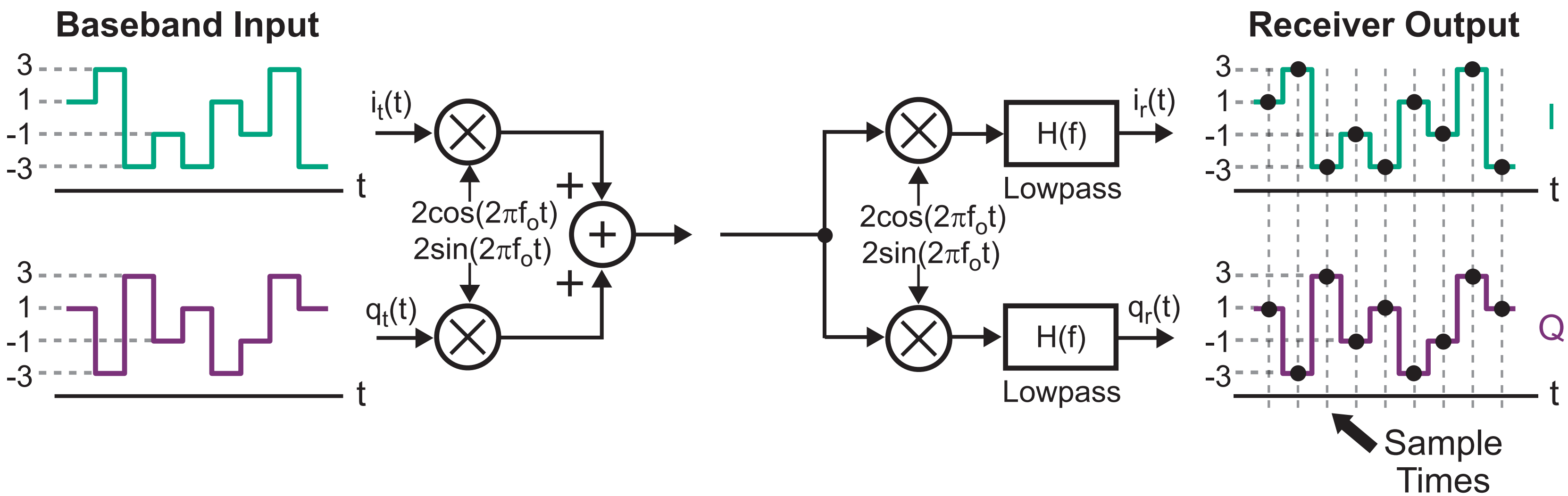
Digital Modulation (Part II)

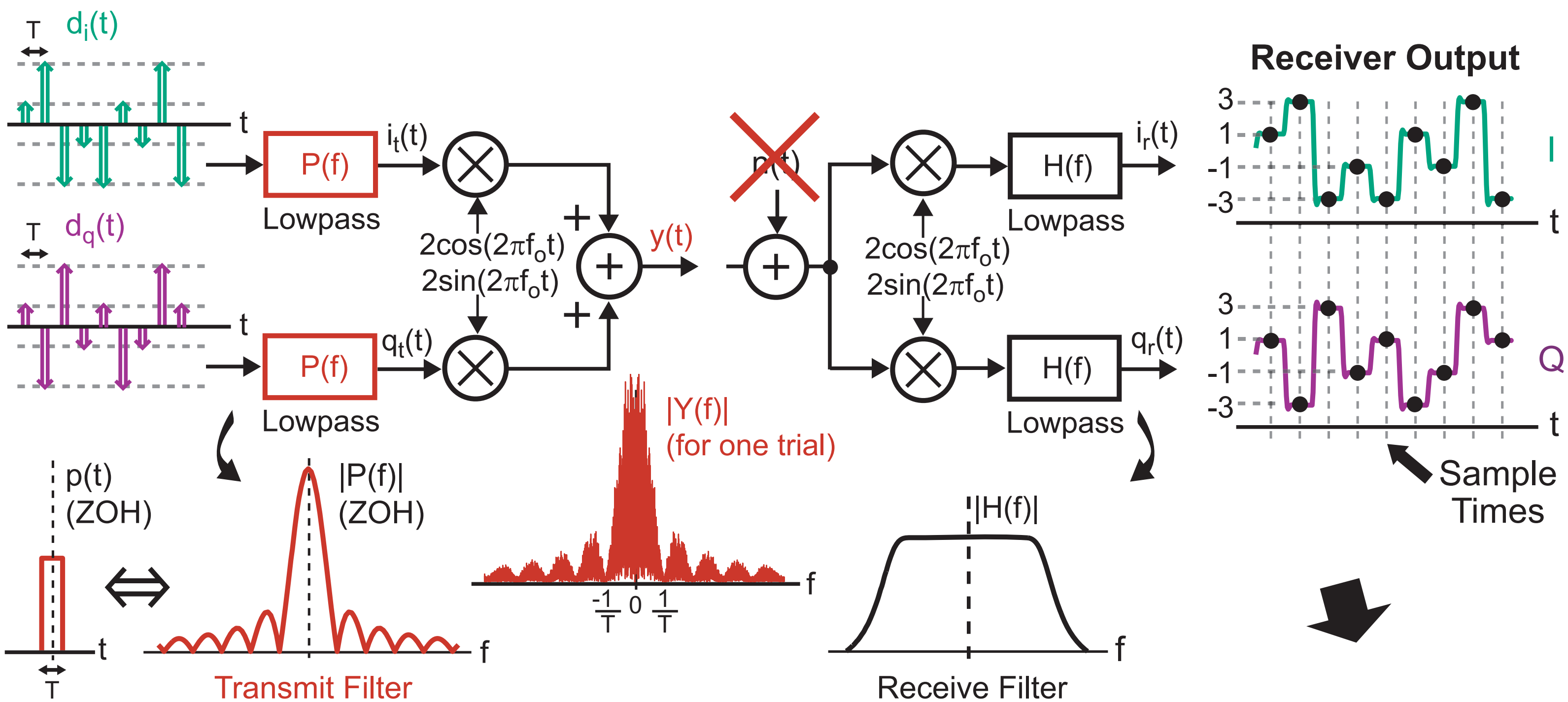
- Receiver noise vs. intersymbol interference (ISI)
- Nyquist Criterion for zero ISI
- (Square-Root) Raised Cosine Filter
- Complex mixing for frequency offset removal

Review of Digital I/Q Modulation

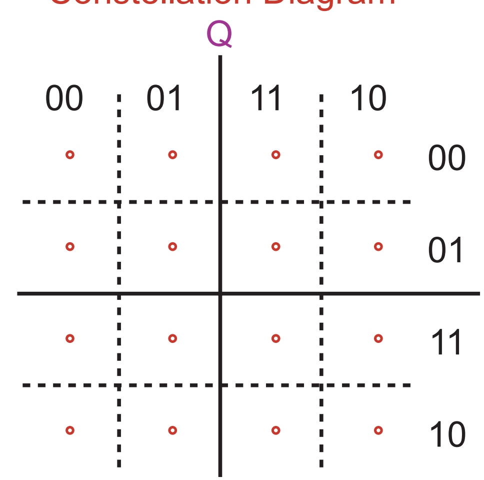


- Leverage *analog communication channel* to send *discrete-valued symbols*
  - Example: send symbol from set {-3,-1,1,3} on both I and Q channels each *symbol period*
- At receiver, sample I/Q waveforms every symbol period
  - Associate each sampled I/Q value with symbols from set {-3,-1,1,3} on both I and Q channels

Transmit and Receiver Filters

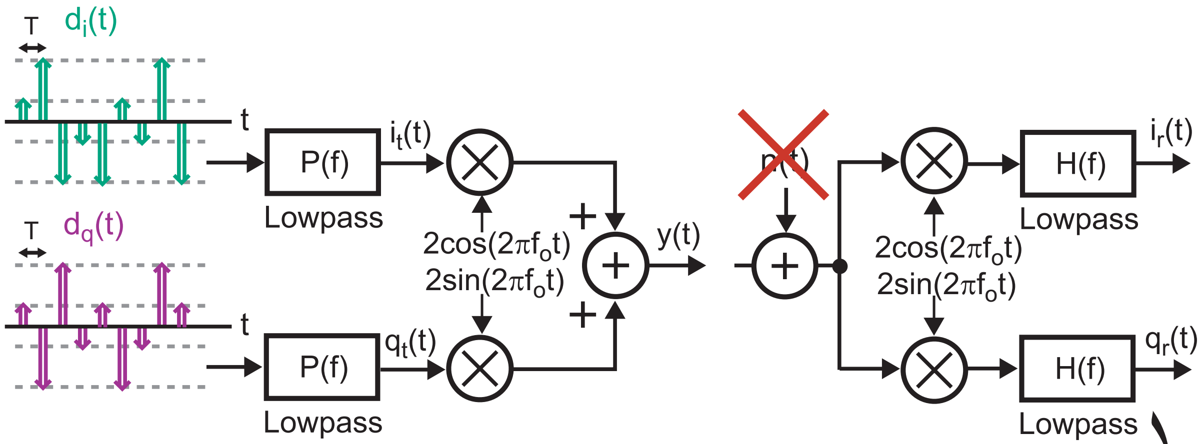


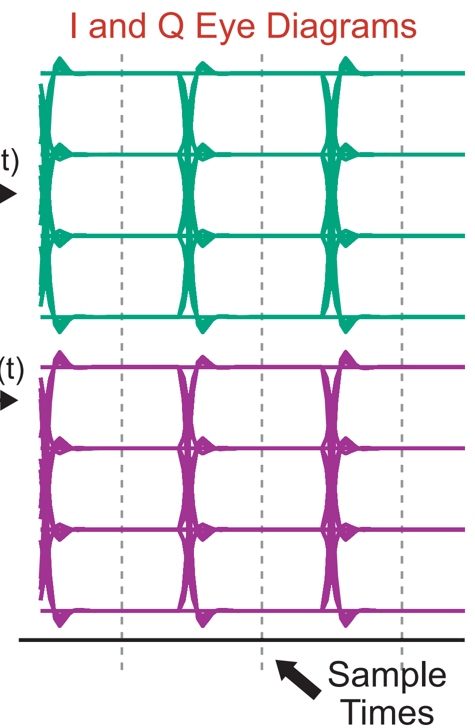
Constellation Diagram

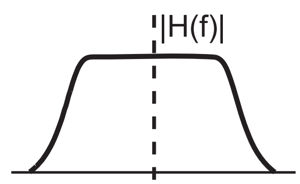


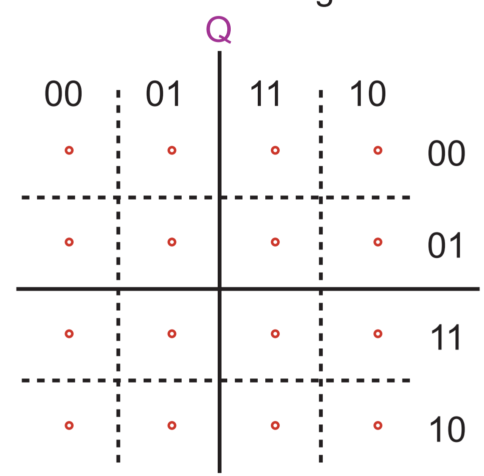
- **Transmit filter examined last time**
  - Tradeoff of transmitted bandwidth vs intersymbol interference (ISI)
- **Receive filter examined this time**
  - Previously assumed to have very wide bandwidth so as not to influence ISI

Tools for ISI Examination



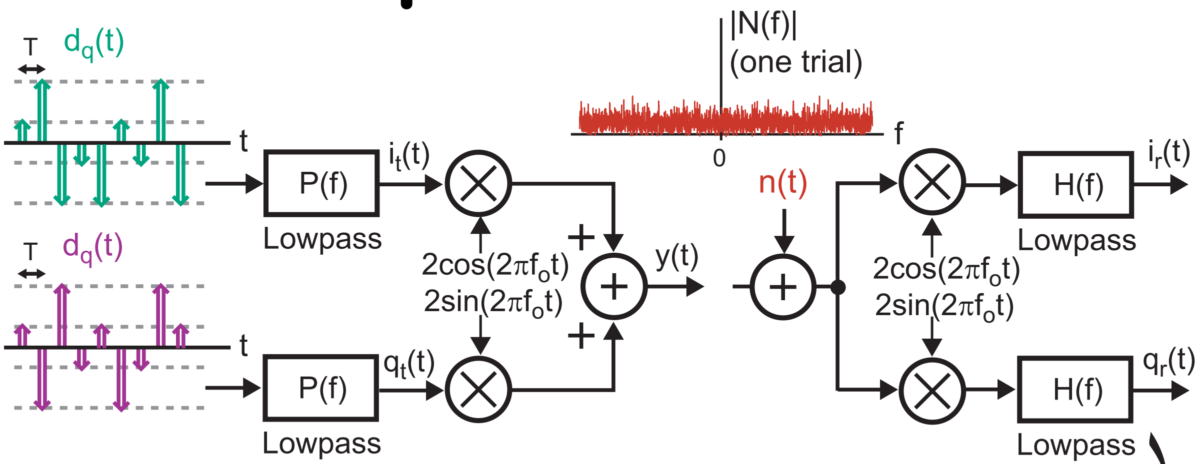


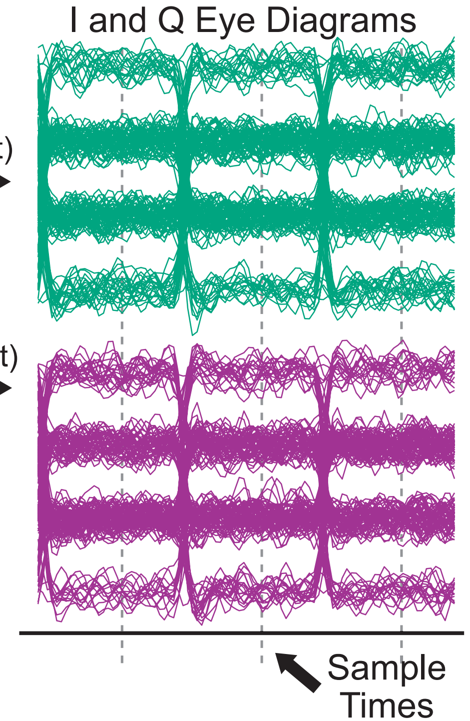


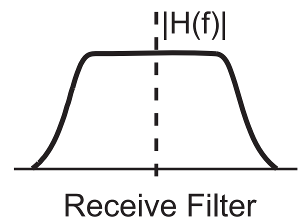


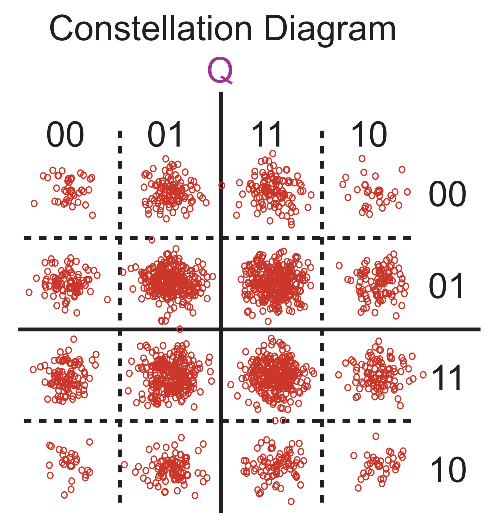
- **Eye diagram**
  - Shows transition behavior between symbols
  - ISI causes closing of eye
- **Constellation diagram**
  - Shows aggregate placement of sampled I/Q values
  - ISI causes spreading of symbol points

Impact of Receiver Noise



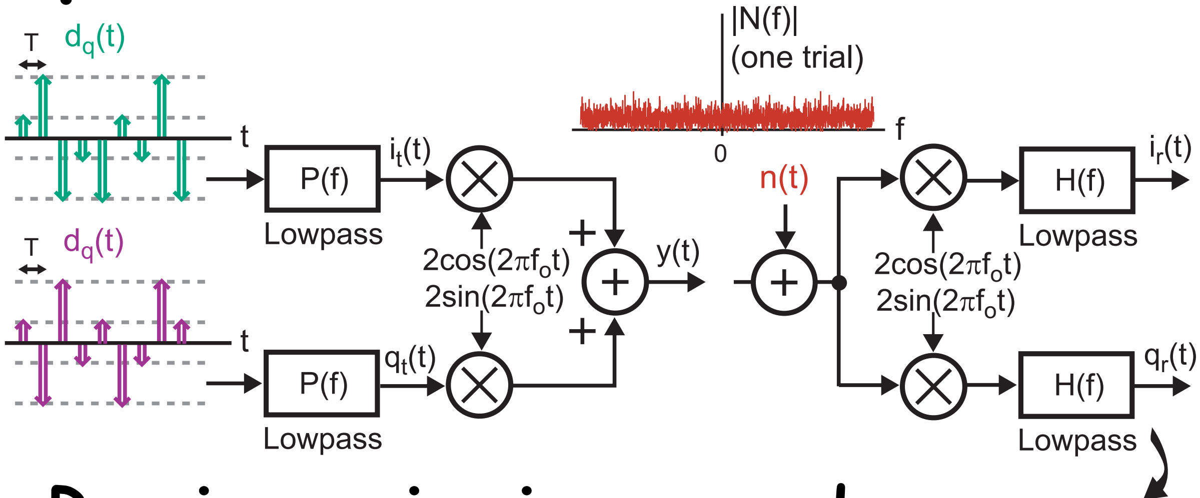


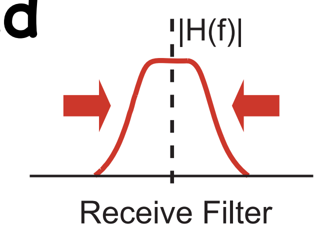


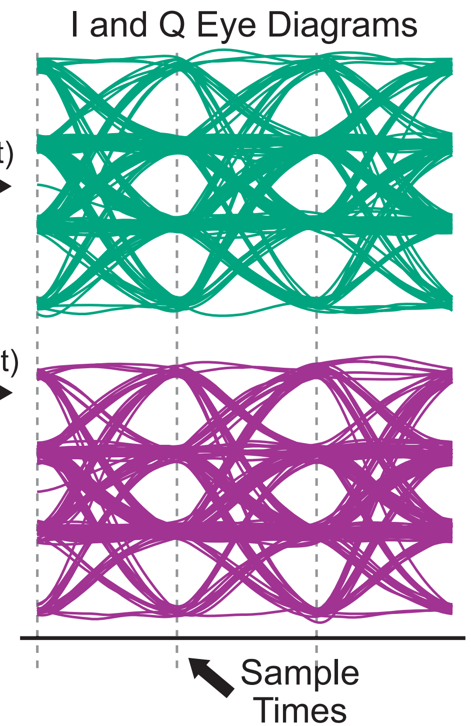


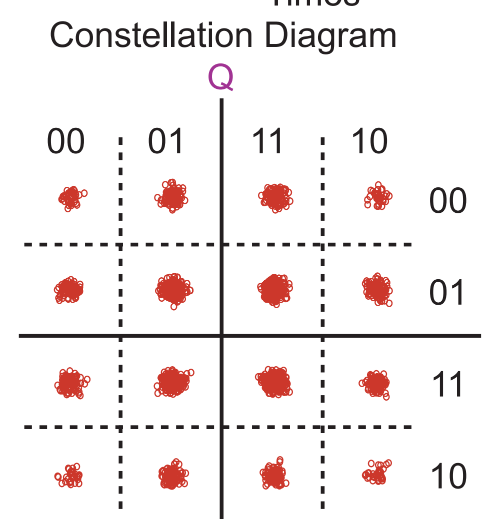
- Receiver noise adds to desired I/Q signals and causes corruption
  - Eye diagram reveals closing of eye
  - Constellation reveals high spread in symbol points
- Key insight: lowering the receive filter bandwidth improves rejection of noise

Impact of Lower Receive Filter Bandwidth





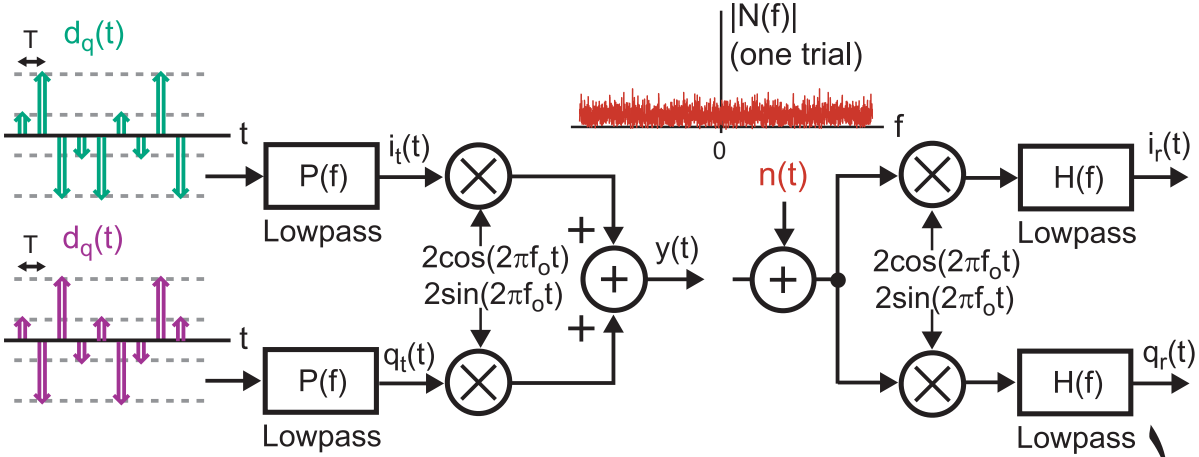


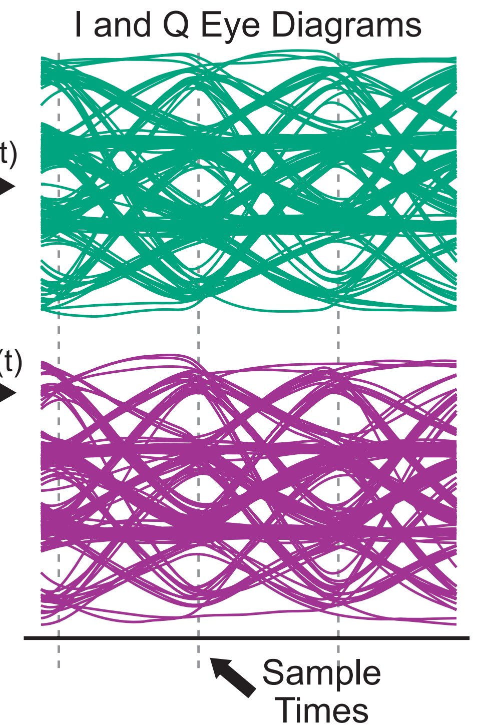


- **Receiver noise is assumed to be *white* (i.e., flat spectrum)**
  - Noise is primarily composed of *thermal noise* in receive circuits
- **Receive filter only passes noise within its *passband***

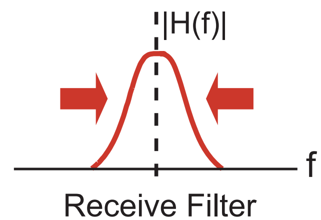
How much can we lower the receive filter bandwidth?

ISI Versus Noise

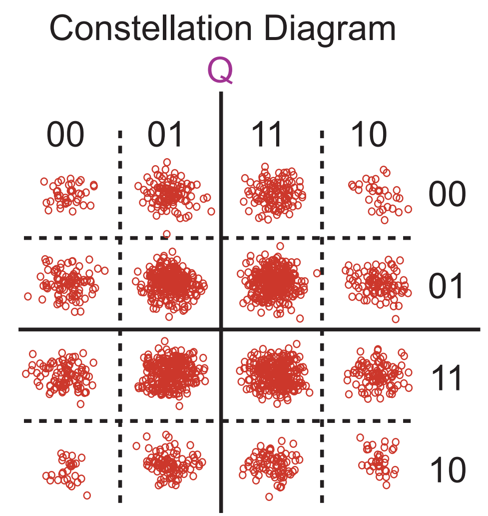




- Lowering receive filter bandwidth too much causes ISI to dominate

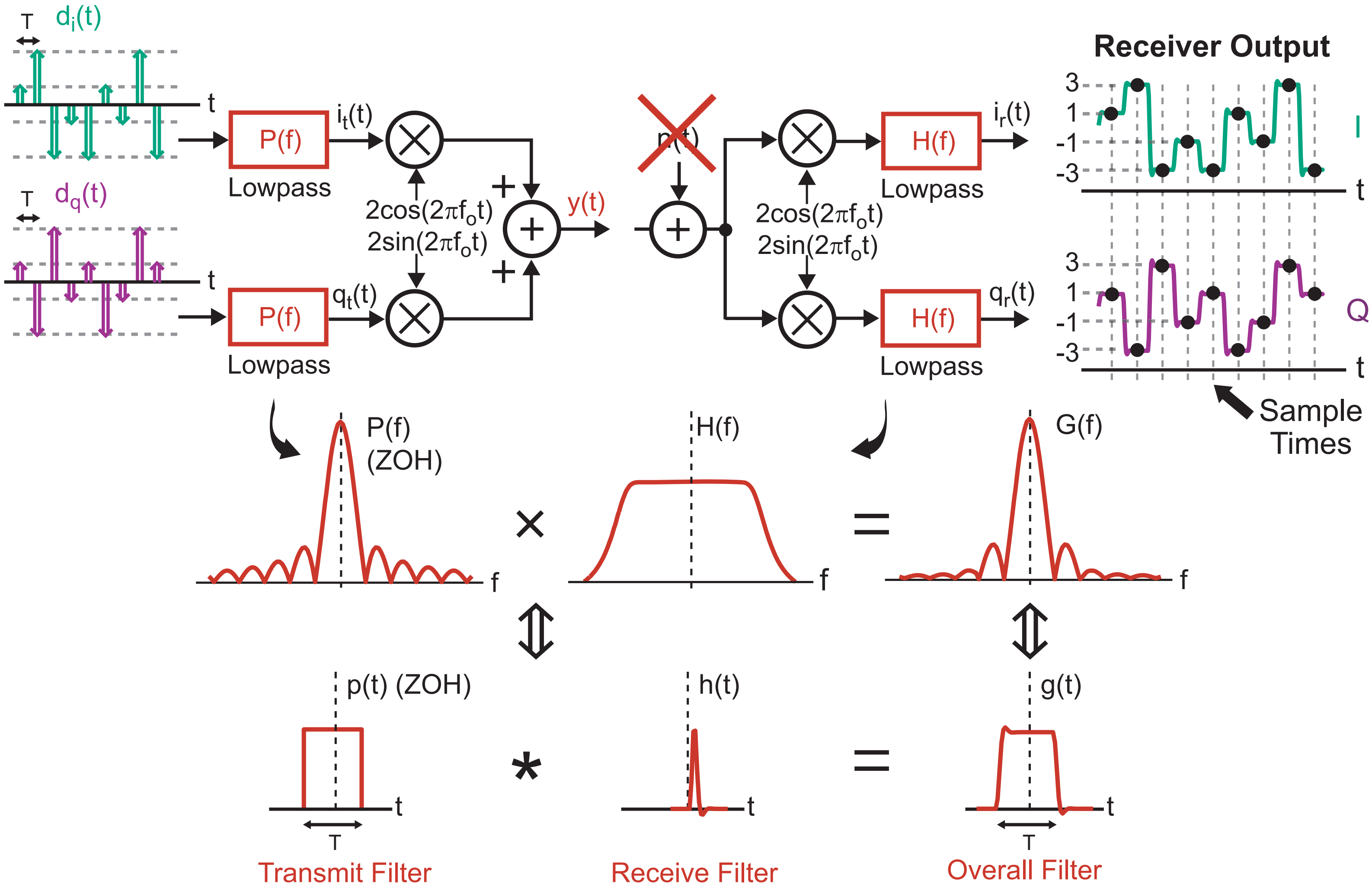


- Selection of receive filter bandwidth involves a tradeoff between ISI and noise
  - Bandwidth too high: high noise
  - Bandwidth too low: high ISI



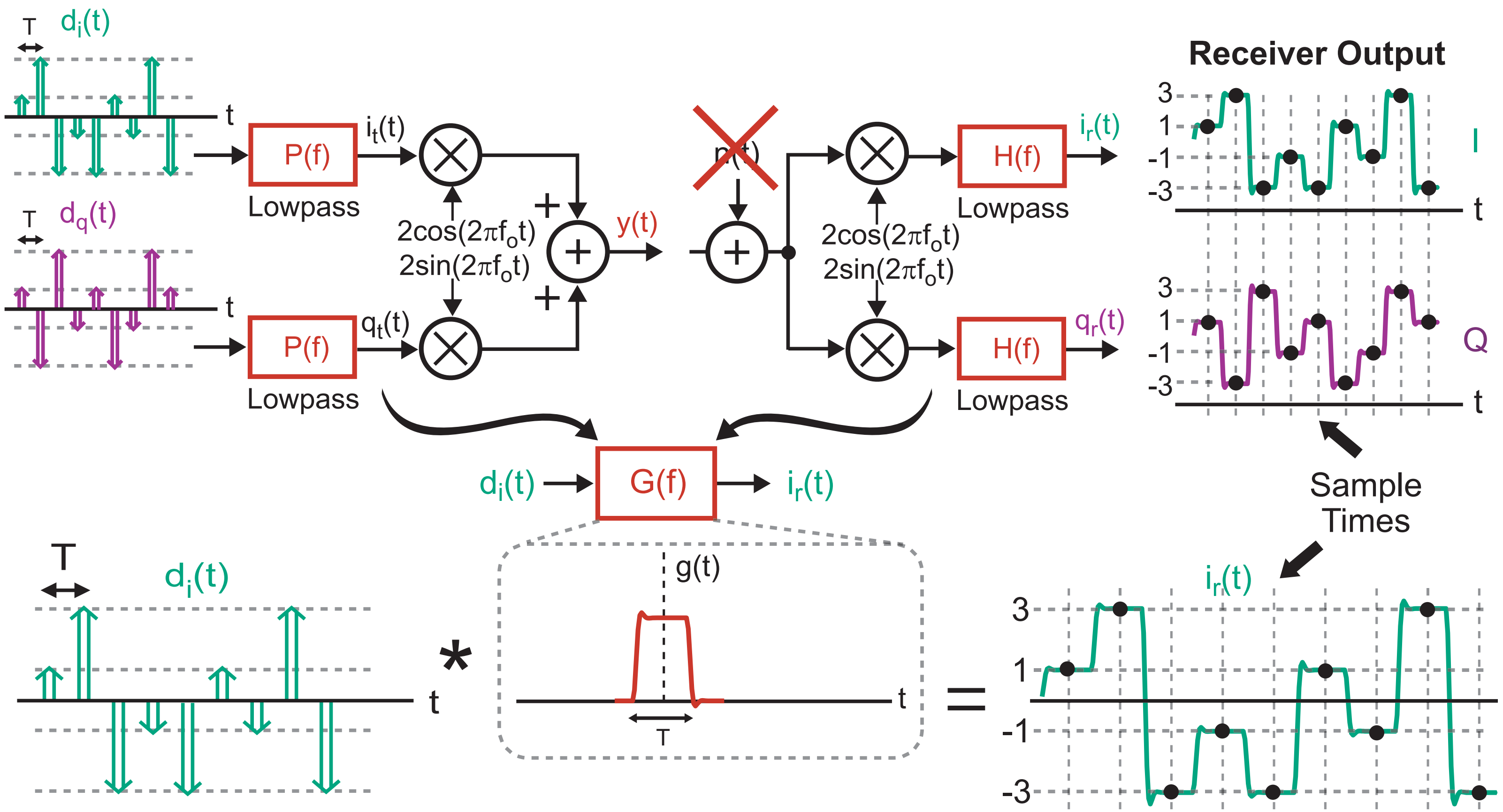
We need to learn more about ISI

Joint Transmit/Receive ISI Analysis



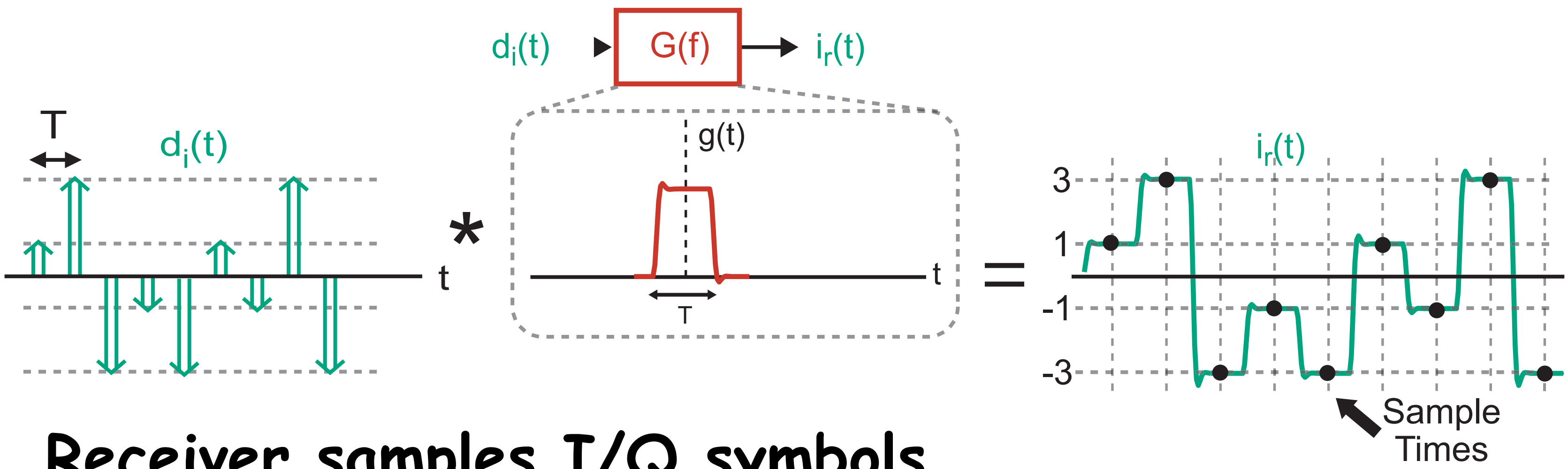
- **ISI influenced by transmit *and* receive filter**
  - **Define *combined* filter response $G(f) = P(f)H(f)$**

Viewing Filtering in the Time Domain

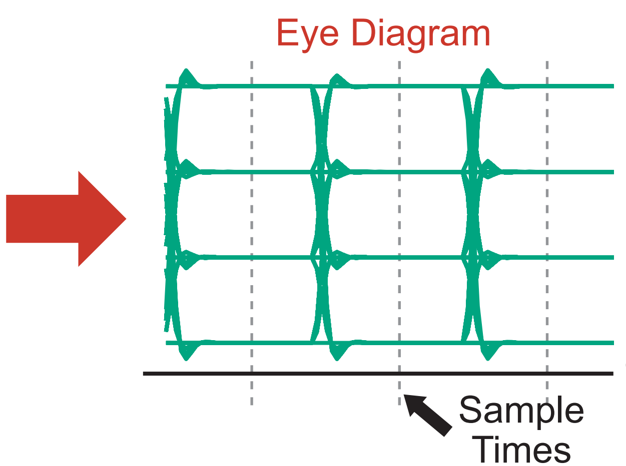


- Filtering operation corresponds to *convolution* in the time domain with *impulse response* (i.e., *g(t)*)
- Time domain view allows us to more directly see impact of overall filter on ISI

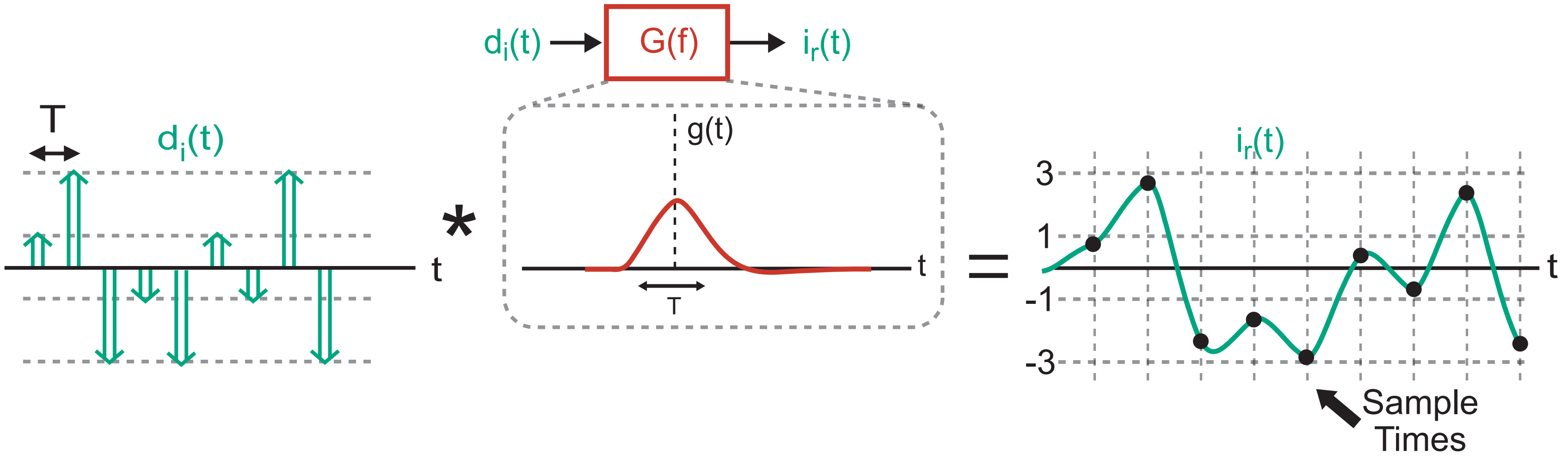
Impulse Response and ISI (High Bandwidth)



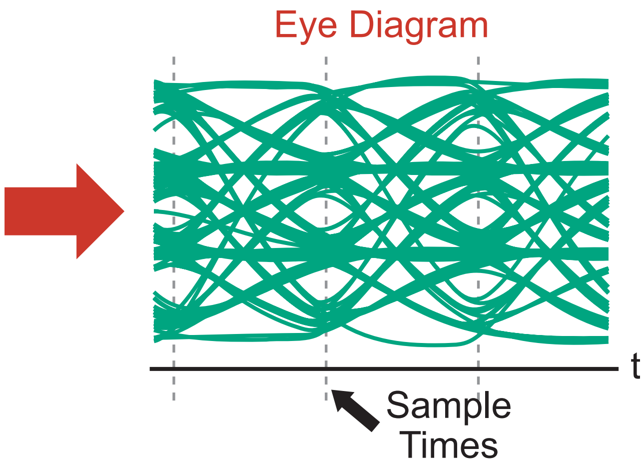
- **Receiver samples I/Q symbols every symbol period**
  - **Achievement of zero ISI requires that each symbol influence only *one* sample at the filter output**
- **Issue: we want lower overall filter bandwidth to reduce transmitter spectrum bandwidth and/or lower receiver noise**
  - **Causes *smoothing* of *g(t)***



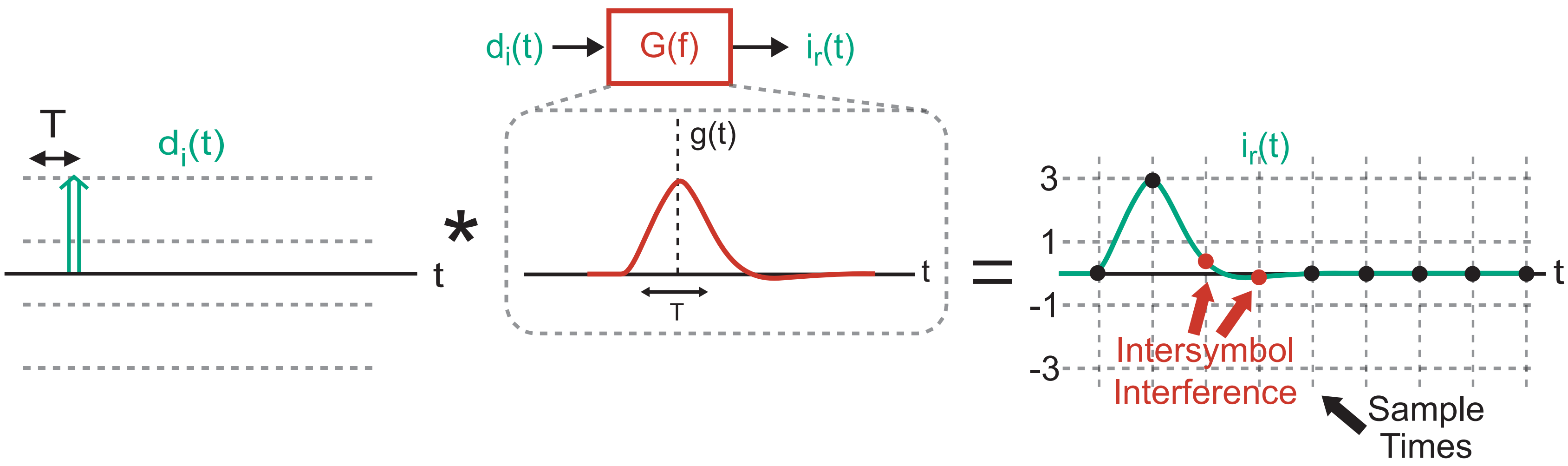
Impulse Response and ISI (Low Bandwidth)



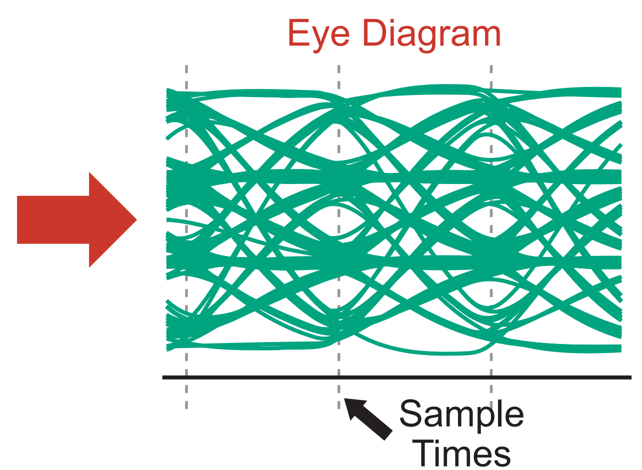
- **Smoothed impulse response has a span longer than one symbol period**
  - **Convolution operation reveals that each symbol impacts filter output at *more* than one sample value**
    - **Intersymbol interference occurs**



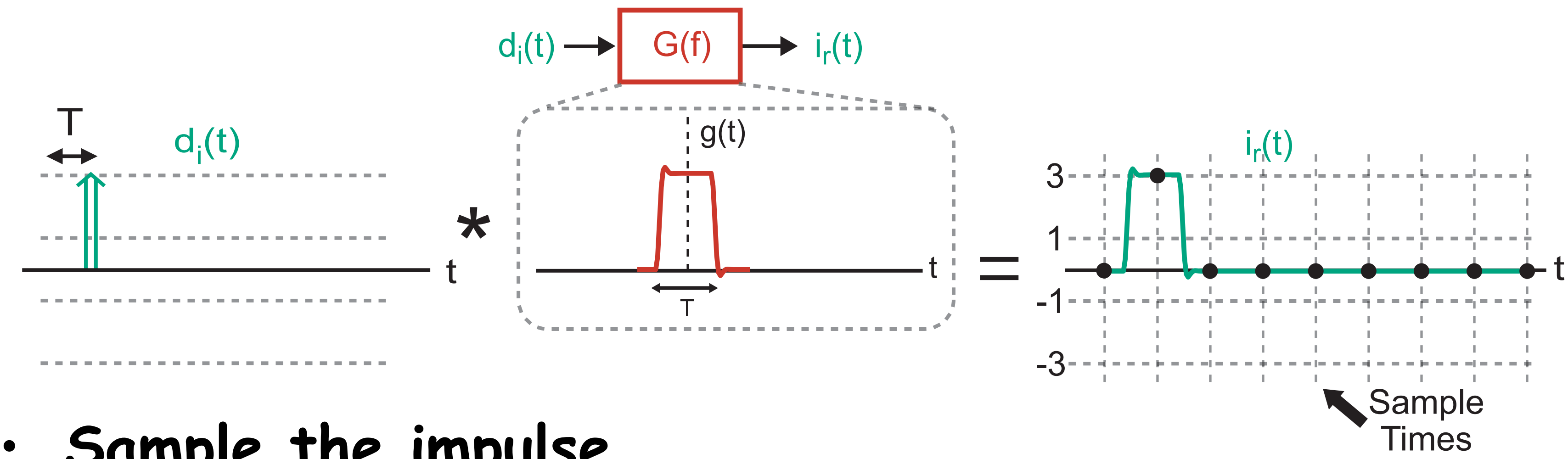
A More Direct View of ISI Issue



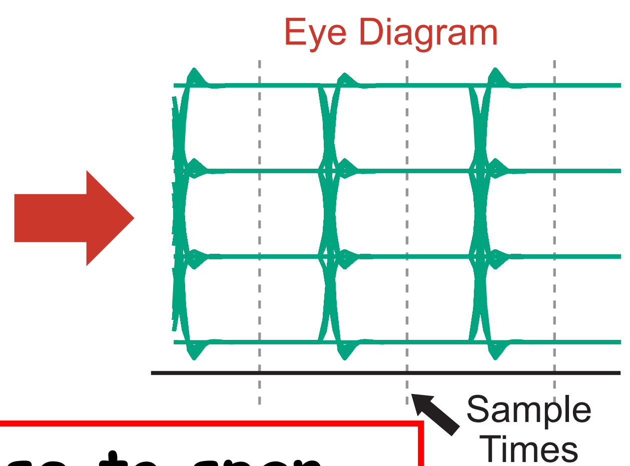
- **Consider impact of just one symbol**
  - **Samples at filter output more clearly show the impact of the one symbol on other sample values**



The Nyquist Criterion for Zero ISI

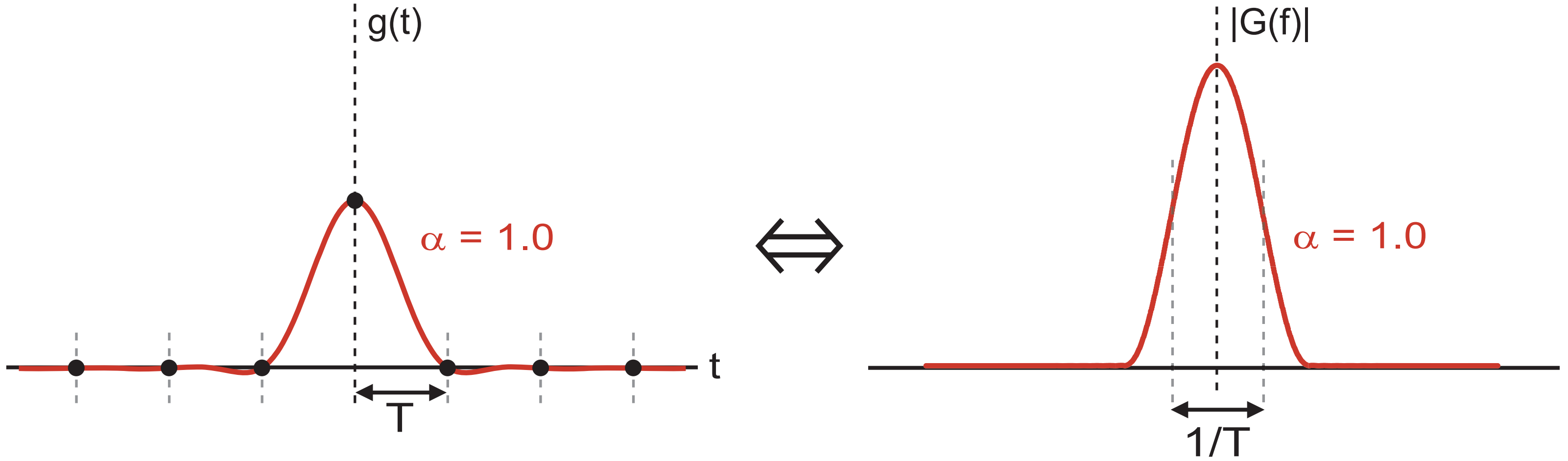


- Sample the impulse response of the overall filter at the symbol period
- Resulting samples must have only *one* non-zero value to achieve zero ISI



Can we *design* impulse response to span more than one symbol period and *still* meet the Nyquist Criterion for zero ISI?

Raised Cosine Filter

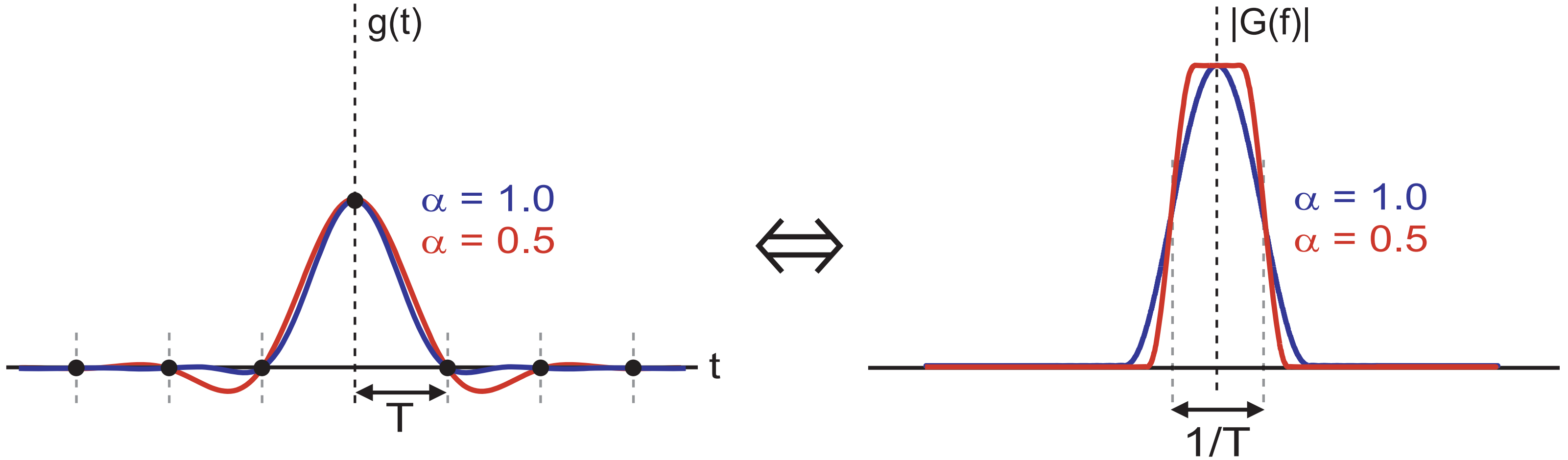


- **Raised cosine filter achieves low bandwidth and zero ISI**
  - **Impulse response spans more than one symbol, but has only *one* non-zero sample value**
  - **Described by function:**

$$g(t) = \frac{\sin(\pi t/T)}{\pi t/T} \frac{\cos(\alpha \pi t/T)}{1 - (2\alpha t/T)^2}$$

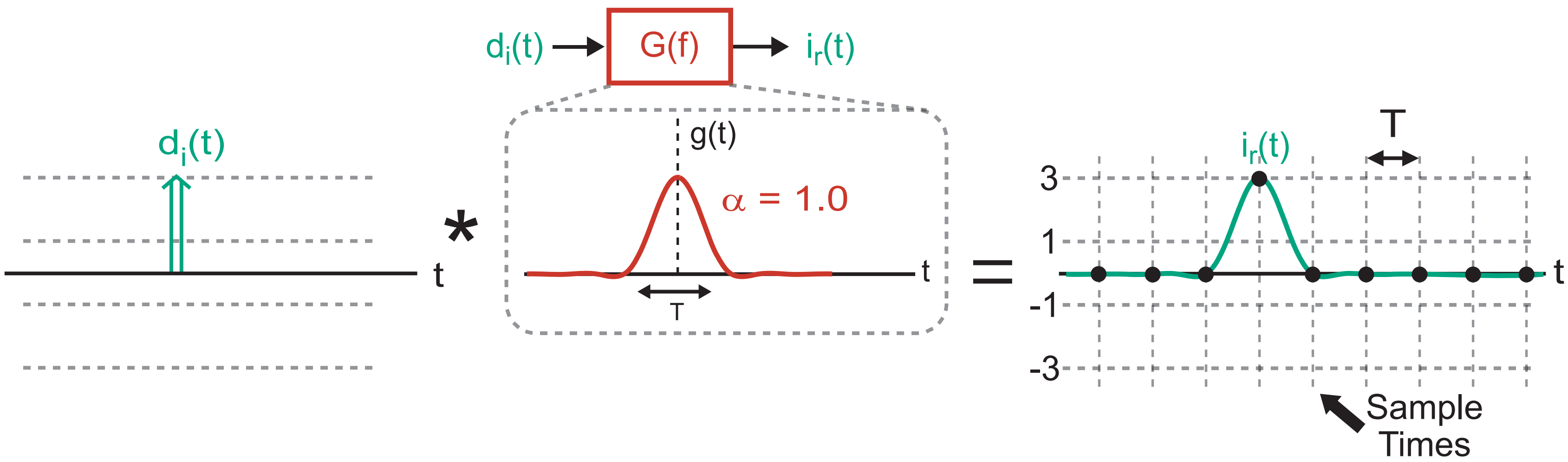
What is the impact of the parameter α?

Impact of α for Raised Cosine Filter

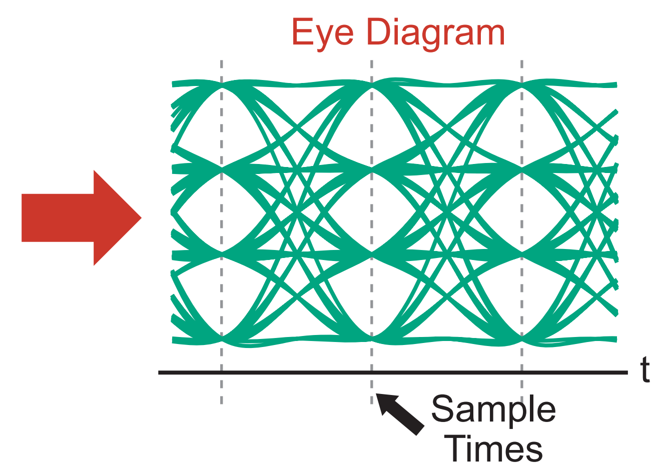


- Parameter α is referred to as the *roll-off* factor of the filter, where $0 \leq \alpha \leq 1$
- Smaller values of α lead to
  - Reduced filter bandwidth
  - Increased duration of the filter impulse response
- Regardless of the value of α, the raised cosine filter allows achieves zero ISI
  - Eye diagrams useful to see impact of α

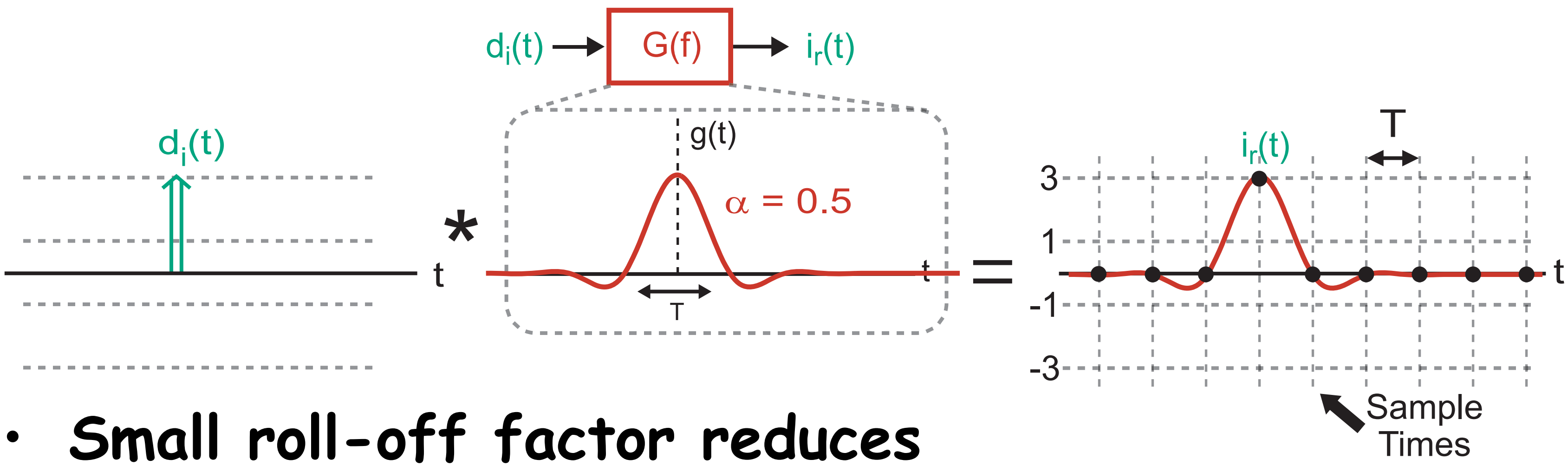
Impact of Large α on Eye Diagram



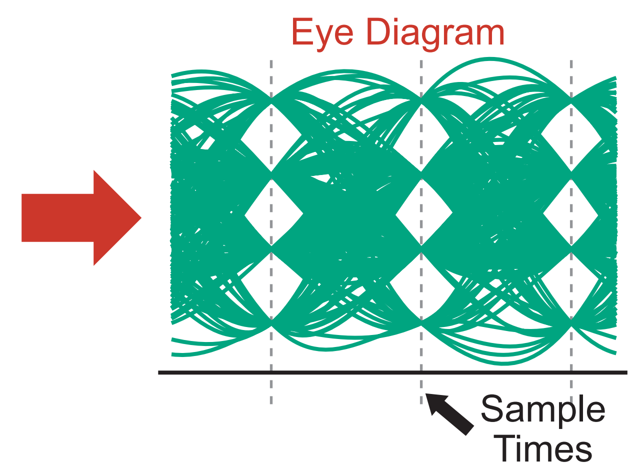
- Large roll-off factor leads to nice, open eye diagram
- Key observation: achievement of zero ISI requires precise placement of sample times
  - Error in placement of sample times leads to substantial ISI!



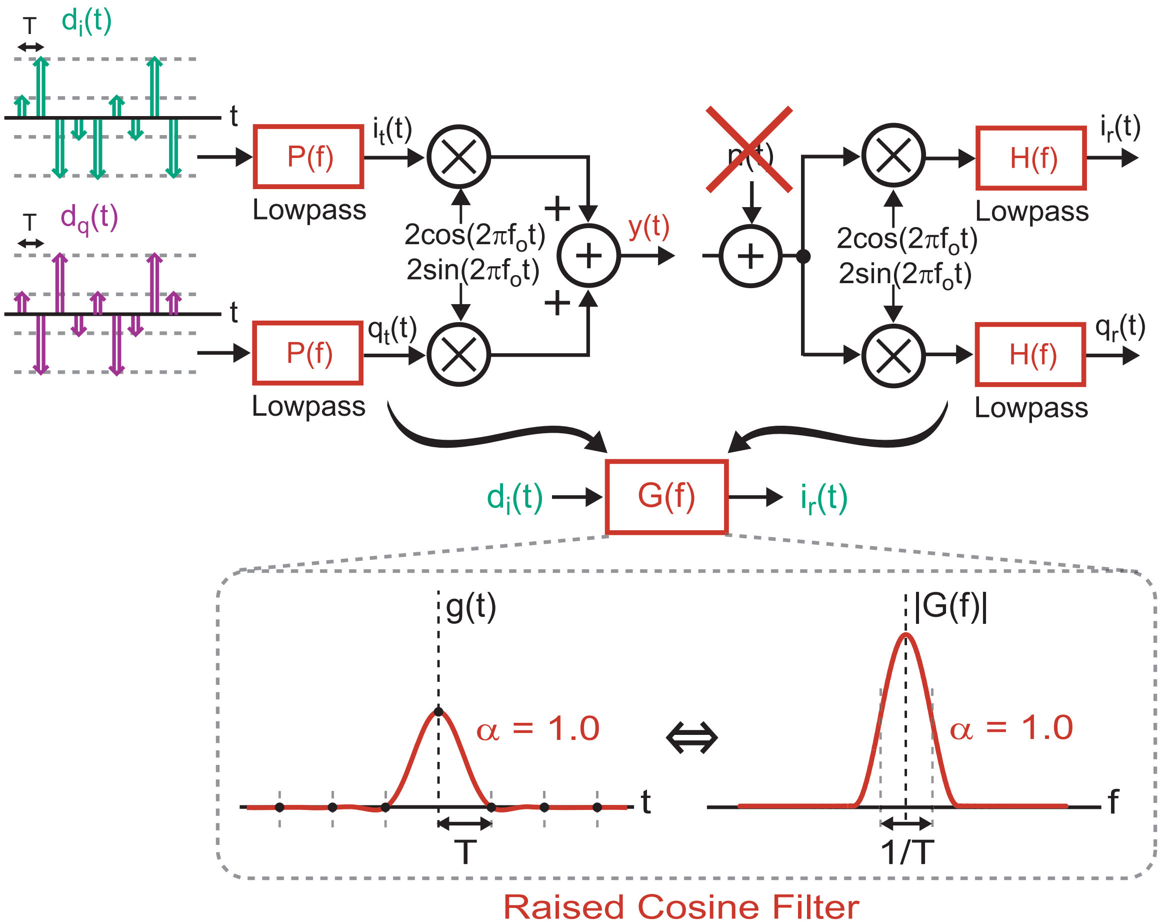
Impact of Small α on Eye Diagram

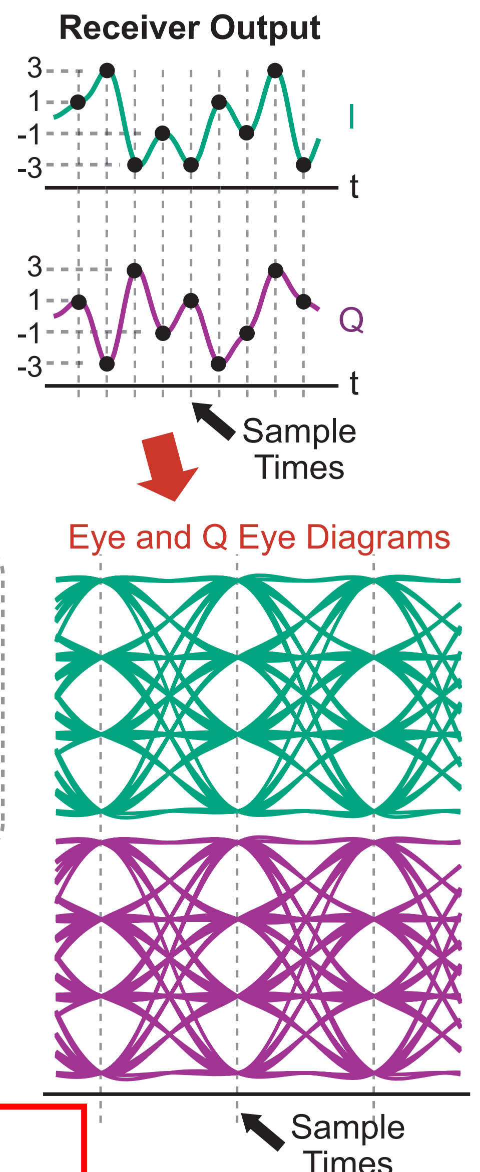


- **Small roll-off factor reduces the filter bandwidth and still allows zero ISI to be achieved**
- **Issue: sensitivity to sample time placement is higher than for large α**
  - **Receiver complexity must be higher to insure high accuracy of sample time placement**



Transmit and Receiver Filter Design



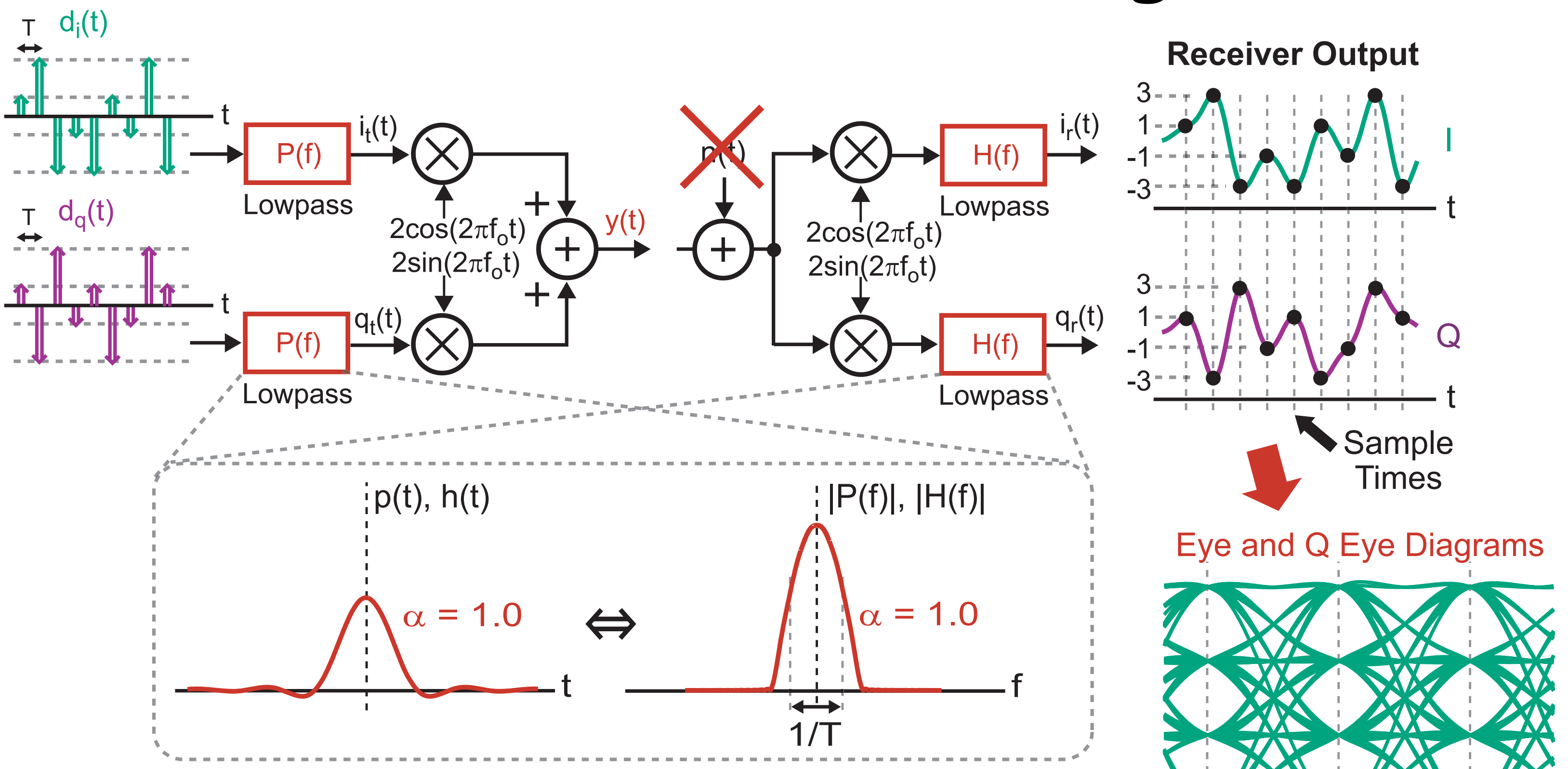


- **Overall response corresponds to**

$$G(f) = P(f)H(f)$$

How do we choose *P(f)* and *H(f)* ?

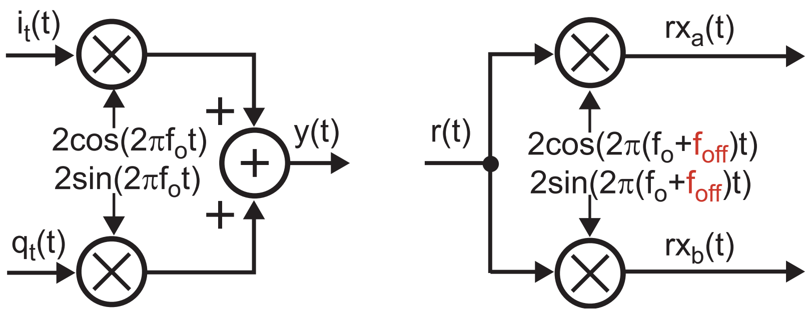
Matched Filter Design



Square-Root Raised Cosine Filter

- Setting H(f) = P(f) yields *matched filter* design
  - Each filter is chosen to be a *Square-Root* Raised Cosine filter
  - 6.011 will discuss in more detail

Motivation for Complex Mixing



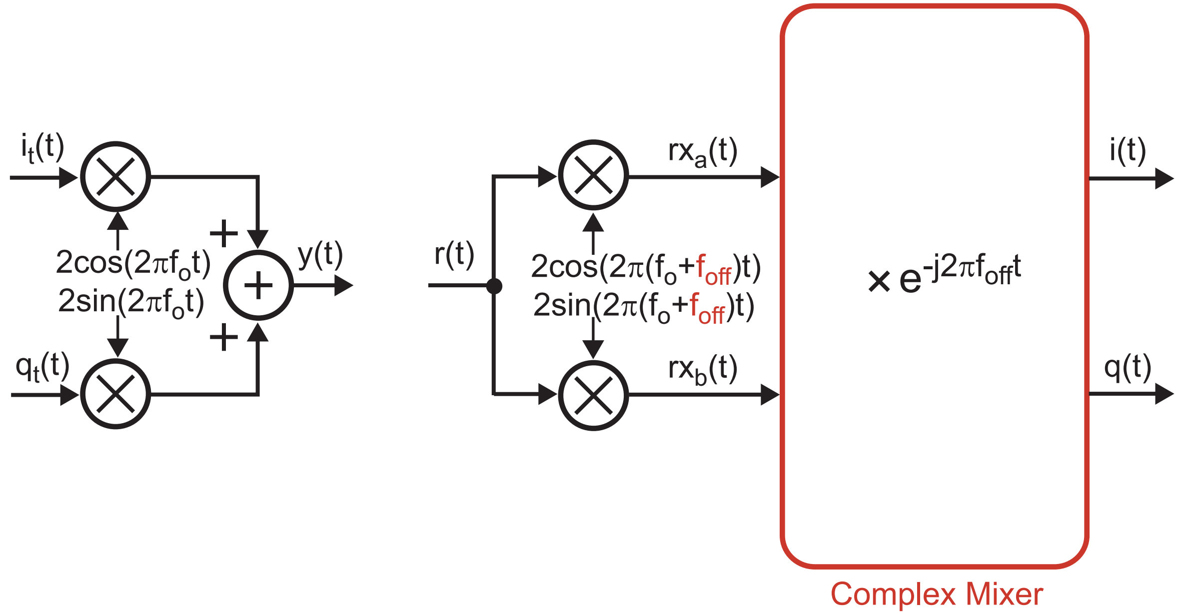
- **Issue: is there a convenient way to undo the receiver frequency offset *after* demodulation by the first set of receive mixers?**
- **Consider looking at received I/Q signals as a *complex* signal:**

$$rx_a(t) + j rx_b(t)$$

$$= r(t)\cos(2\pi(f_o + f_{off})t) + j(r(t)\sin(2\pi(f_o + f_{off}t))$$

$$= r(t)e^{j2\pi(f_o+f_{off})t} = \boxed{r(t)e^{j2\pi f_o t}e^{j2\pi f_{off} t}}$$

Complex Mixer Simplifies Offset Removal



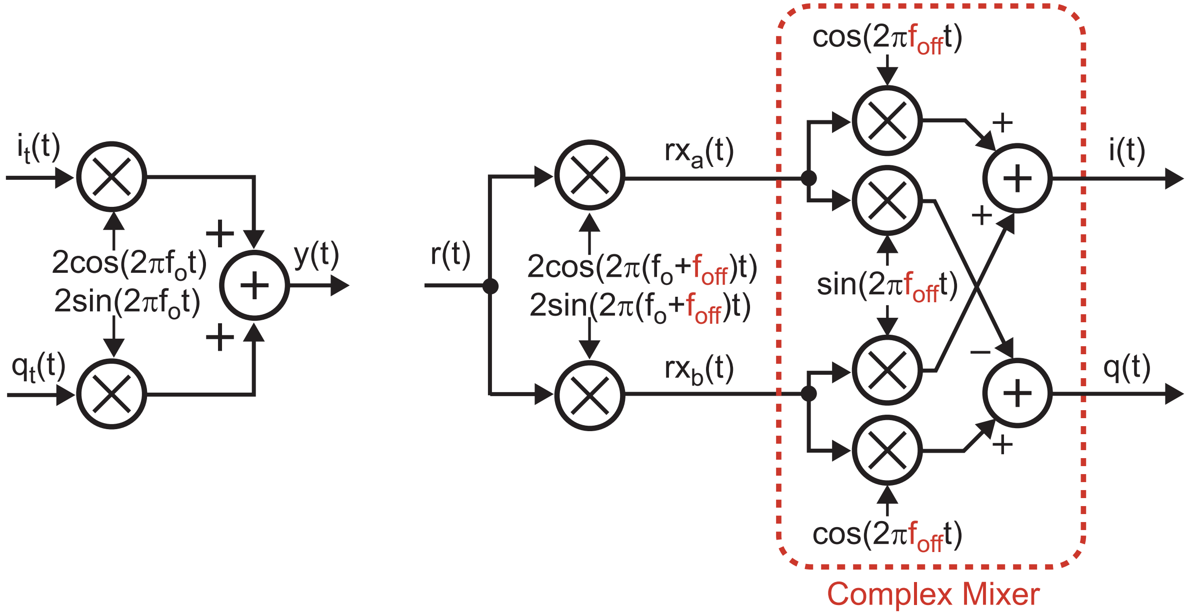
- **Simply multiply input by complex exponential**
  - **Output will also be a complex signal**

$$i(t) + jq(t) = (rx_a(t) + jrx_b(t))e^{-j2\pi f_{off} t}$$

$$= (r(t)e^{j2\pi f_o t}e^{j2\pi f_{off} t})e^{-j2\pi f_{off} t} = r(t)e^{j2\pi f_o t}$$

$$\Rightarrow \boxed{i(t) = r(t)\cos(2\pi f_o t) \qquad q(t) = r(t)\sin(2\pi f_o t)}$$

Practical Implementation of Complex Mixer



- **Complex mixing achieved with four *real* mixers**
  - **Add/subtract outputs appropriately**

$$(rx_a(t) + j rx_b(t))e^{-j2\pi f_{off} t}$$

$$= (rx_a(t) + j rx_b(t))(\cos(2\pi f_{off} t) - j\sin(2\pi f_{off} t))$$

$$= \boxed{\begin{array}{l} rx_a(t)\cos(2\pi f_{off} t) + rx_b(t)\sin(2\pi f_{off} t) + \\ j(rx_b(t)\cos(2\pi f_{off} t) - rx_a(t)\sin(2\pi f_{off} t)) \end{array}}$$

Summary

- Receive filter design involves a tradeoff between noise and ISI
  - Higher bandwidth leads to higher noise
  - Lower bandwidth leads to higher ISI
- Zero ISI can be achieved if Nyquist Criterion is met
  - Choose transmit and receive filters to both be Square-Root Raised Cosine Filters
- Complex mixing offers a convenient way of removing frequency offset
  - We will make use of this in Lab 5
- Next lecture: further examine impact of noise in digital modulation