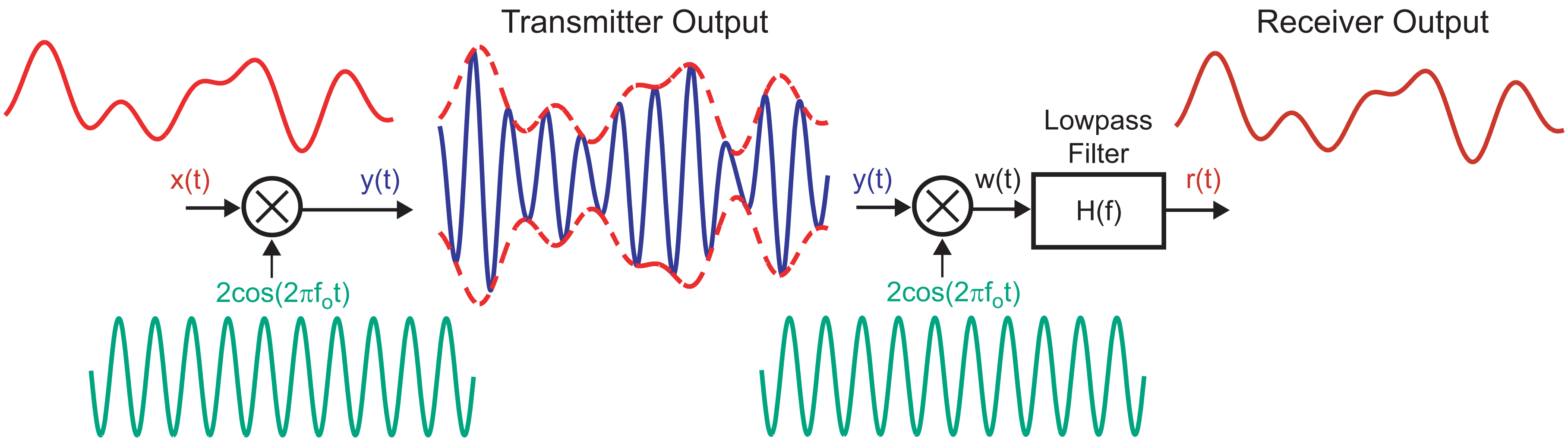
I/Q Modulation and RC Filtering

- Issues with coherent modulation
- Analog I/Q modulation principles
- RC networks as continuous-time filters
- Differentiation property of Fourier Transform

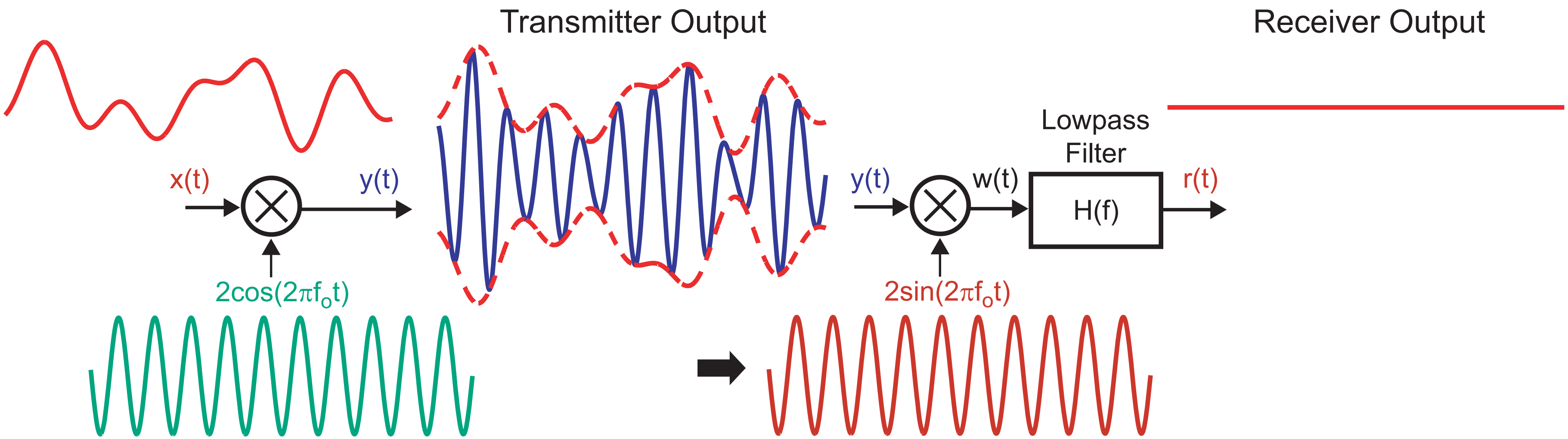
AM Modulation and Demodulation



- **Multiplication (i.e., *mixing*) operation shifts in frequency**
  - **Also creates undesired high frequency components at receiver**
- **Lowpass filtering passes only the desired *baseband* signal at receiver**

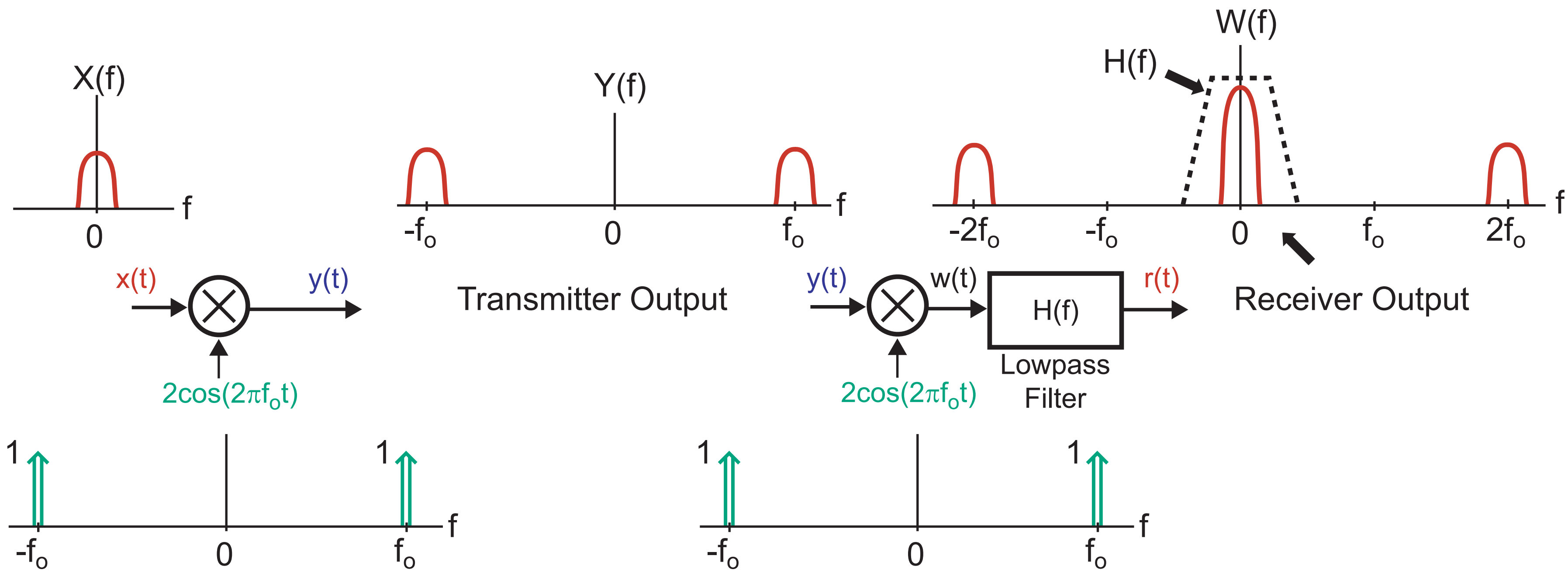
What can go wrong here?

Impact of 90 Degree Phase Shift



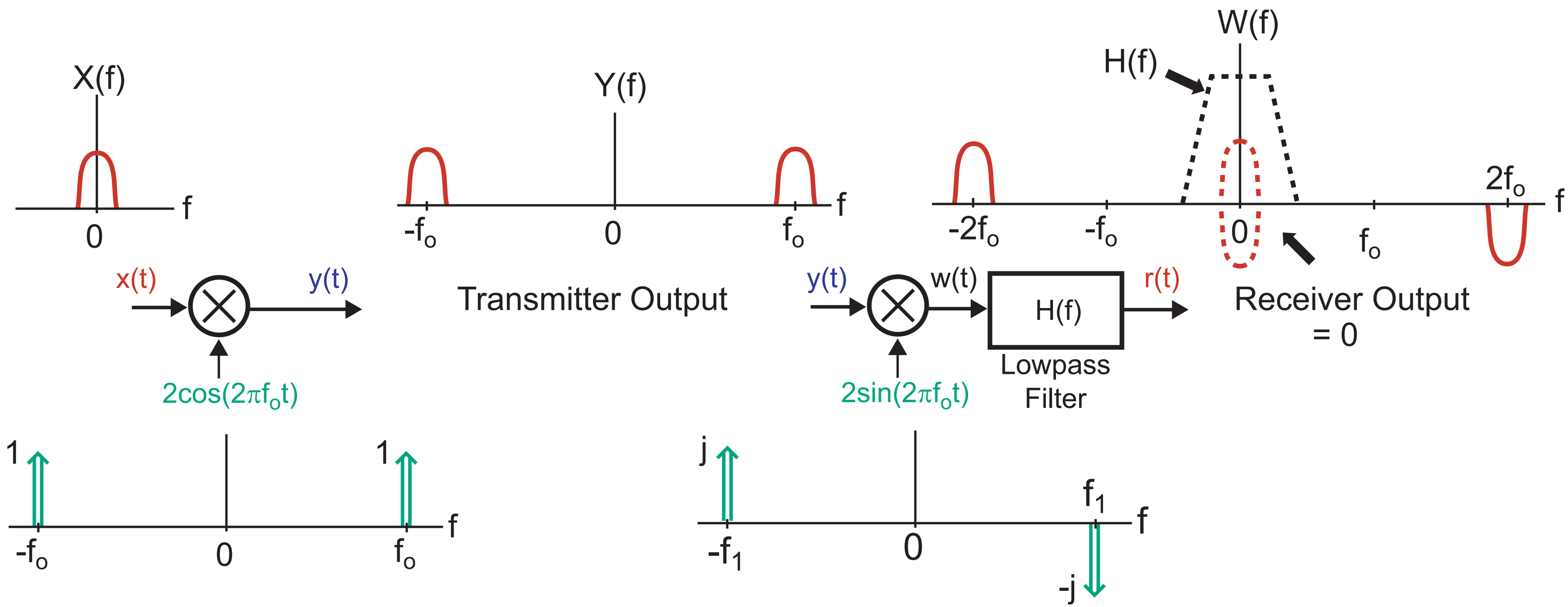
- If receiver cosine wave turns into a sine wave, we suddenly receive no baseband signal!
  - We apparently need to synchronize the phase of the transmitter and receiver *local oscillators*
    - This is called *coherent* demodulation
- Some key questions:
  - How do we analyze this issue?
  - What would be the impact of a small *frequency* offset?

Frequency Domain Analysis



- **When transmitter and receiver local oscillators are matched in phase:**
  - **Demodulated signal *constructively* adds at baseband**

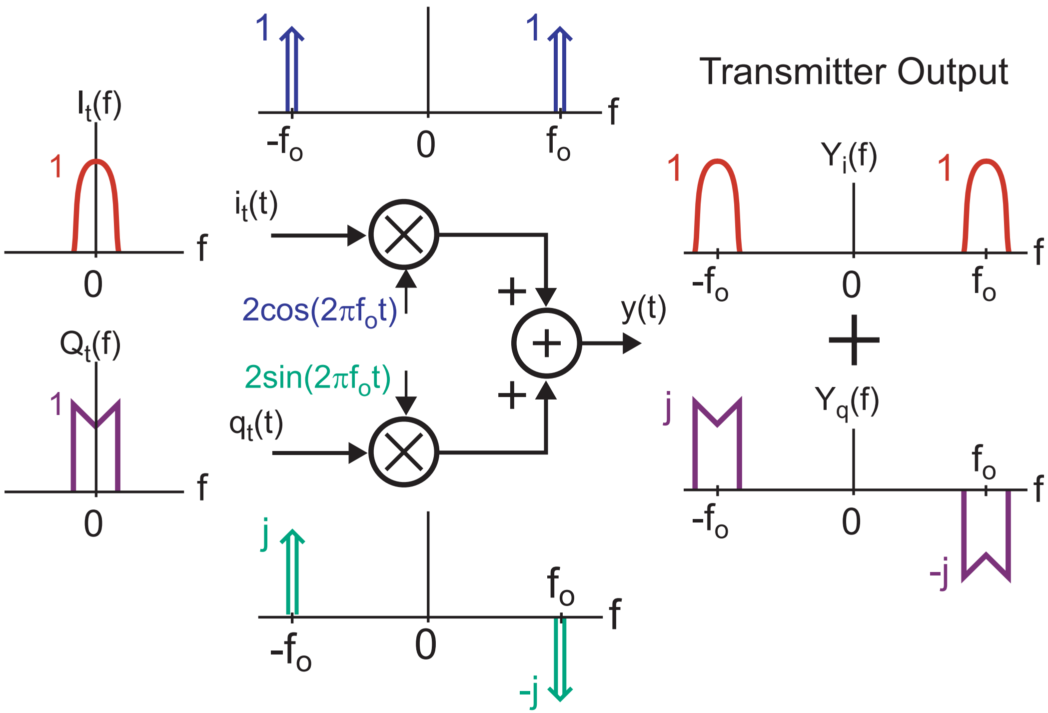
Impact of 90 Degree Phase Shift



- **When transmitter and receiver local oscillators are 90 degree offset in phase:**
  - **Demodulated signal *destructively* adds at baseband**

What would happen with a small frequency offset?

I/Q Modulation

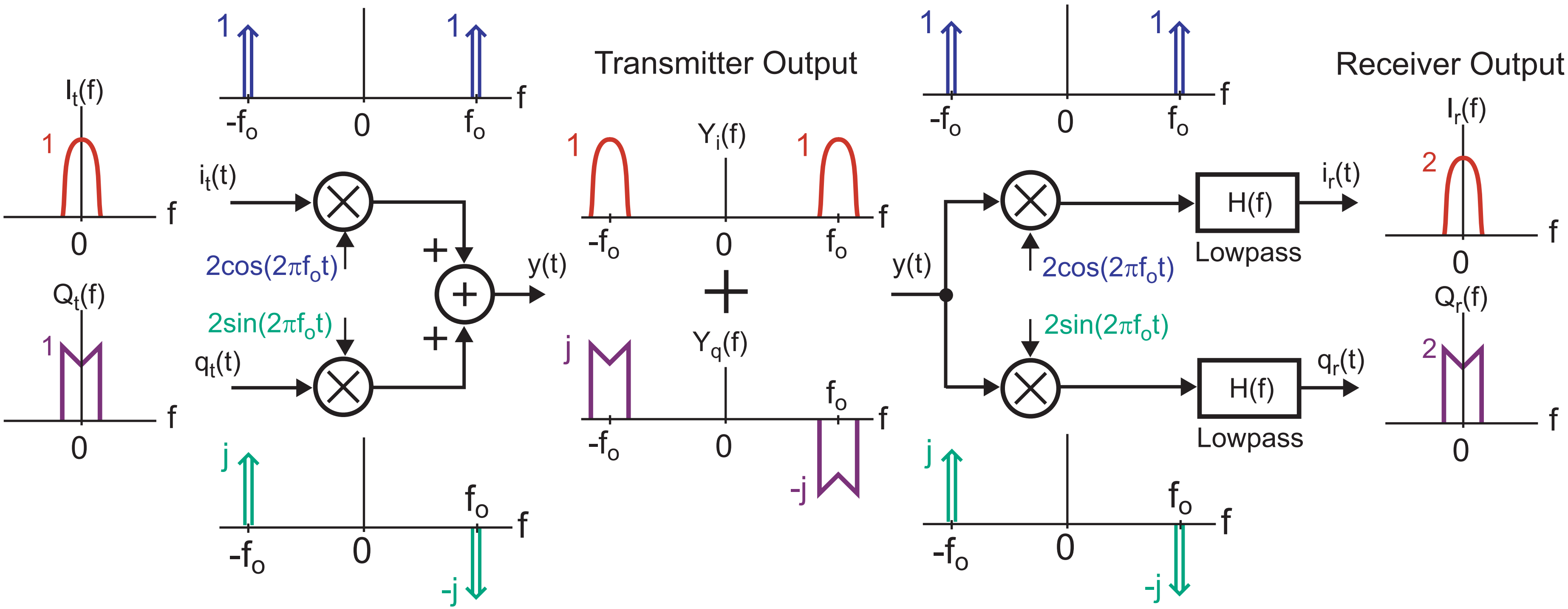


I stands for *in-phase* component

Q stands for *quadrature* component

- Consider modulating with both a cosine and sine wave and then adding the results
  - This is known as I/Q modulation
- The I/Q signals occupy the same frequency band, but one is *real* and one is *imaginary*
  - We will see that we can recover *both* of these signals

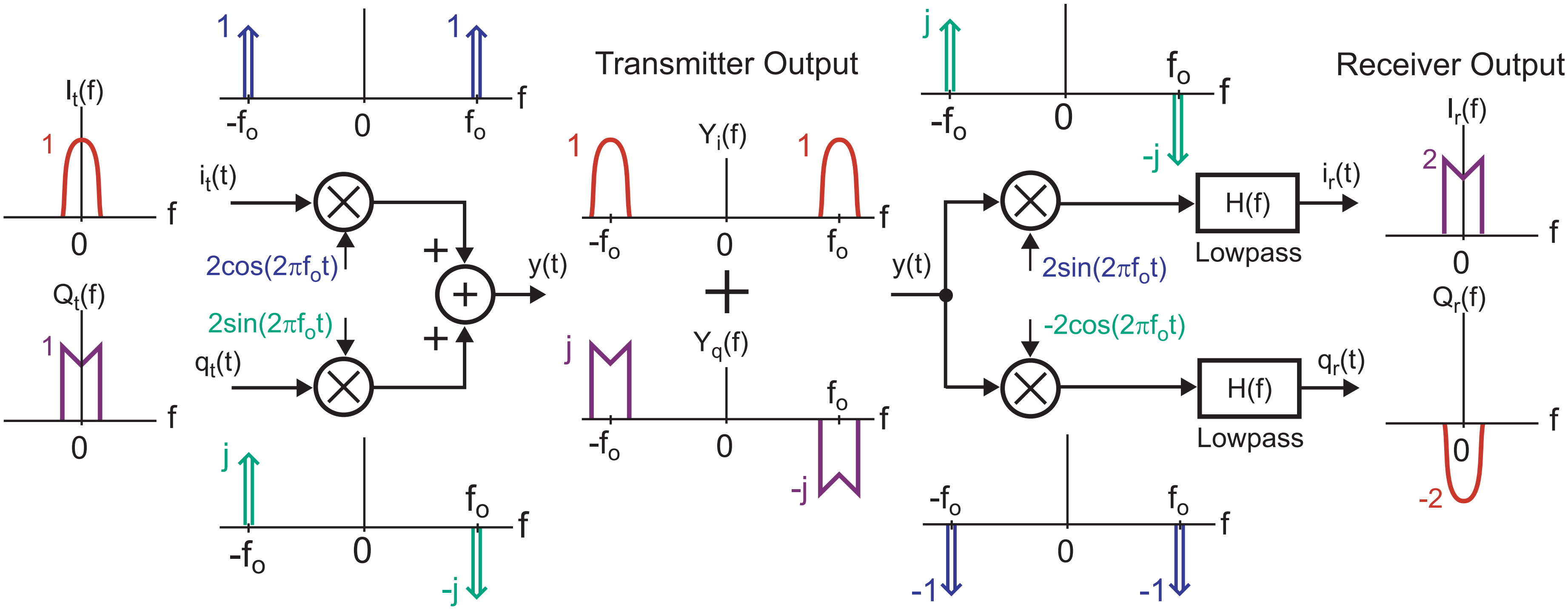
I/Q Demodulation



- Demodulate with *both* a cosine and sine wave
  - Both I and Q channels are recovered!
- I/Q modulation allows twice the amount of *information* to be sent compared to basic AM modulation with same *bandwidth*

What can go wrong here?

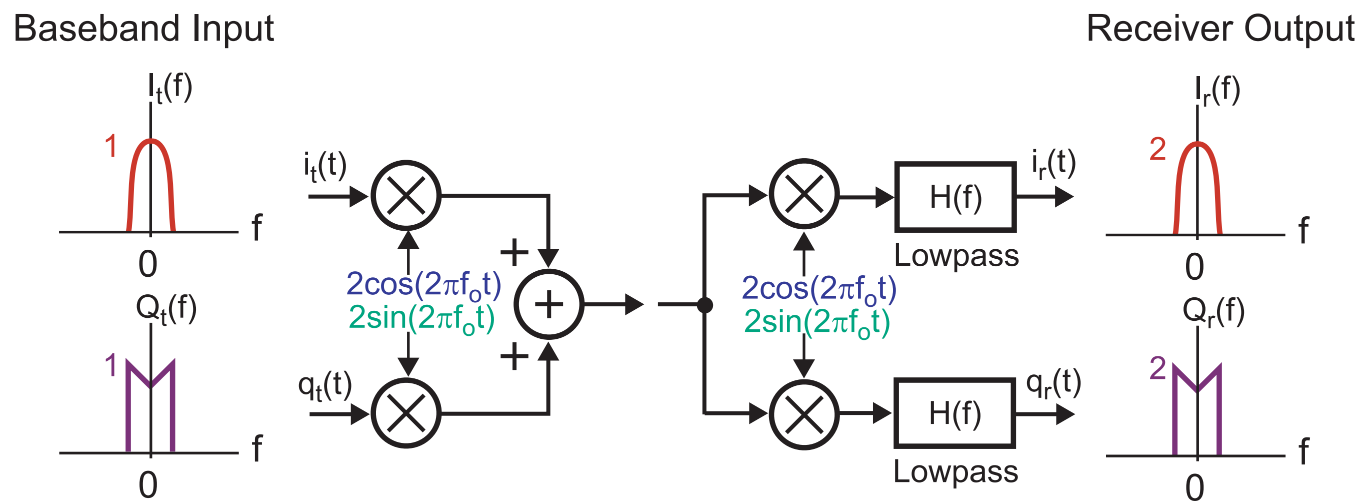
Impact of 90 Degree Phase Shift



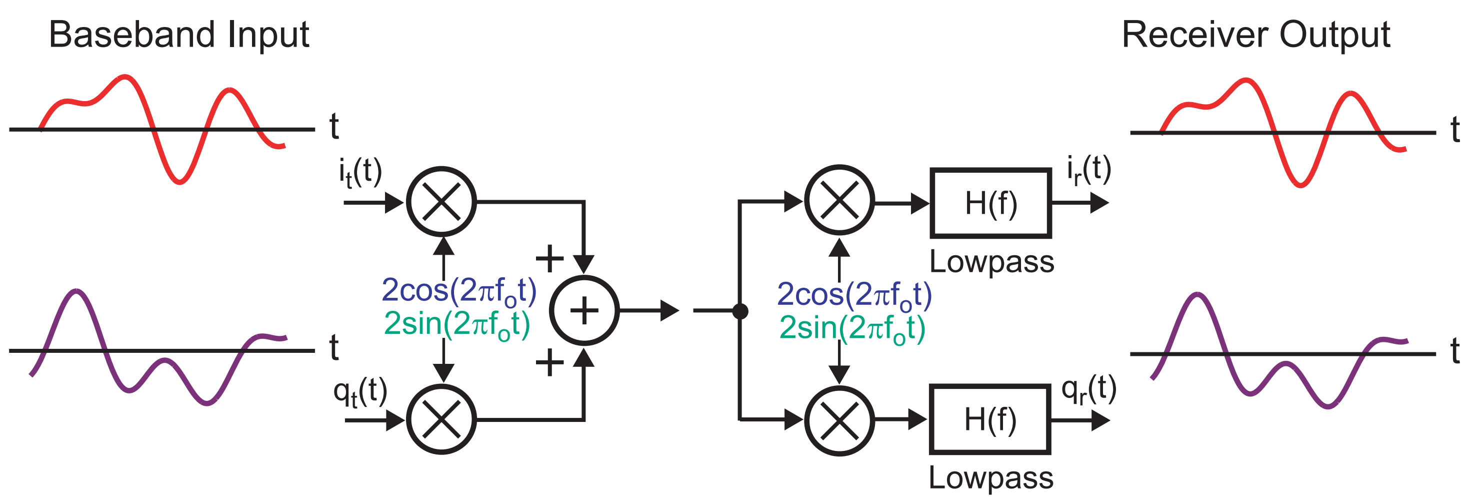
- I and Q channels get swapped at receiver
  - Key observation: no *information* is lost!
- Questions
  - What would happen with a *small* frequency offset?
  - What would happen with a *large* frequency offset?

Summary of *Analog* I/Q Modulation

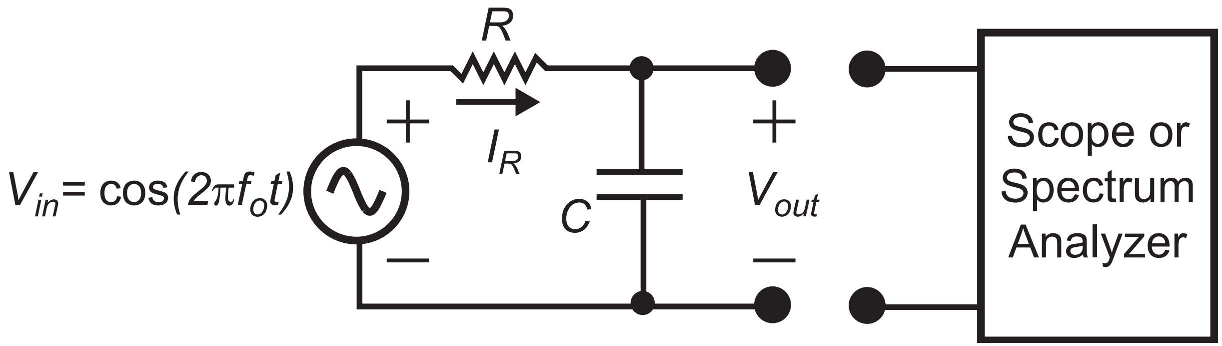
- **Frequency domain view**



- **Time domain view**



RC Filter



- **Analyze by first deriving a differential equation relating output and input voltages**

$$I_R(t) = \frac{V_{in}(t) - V_{out}(t)}{R} = C\frac{dV_{out}(t)}{dt}$$

- **The filter frequency response is defined as**

$$H(f) = \frac{V_{out}(f)}{V_{in}(f)}$$

- **The output voltage corresponds to a scaled and phase shifted version of the input cosine wave**

$$V_{out}(t) = |H(f_o)|\cos(2\pi f_o t + \angle H(f_o))$$

Differentiation Property of FT

Fourier Transform Definition

$$x(t) \Leftrightarrow X(f) \quad \begin{cases} x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df \\ X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt \end{cases}$$

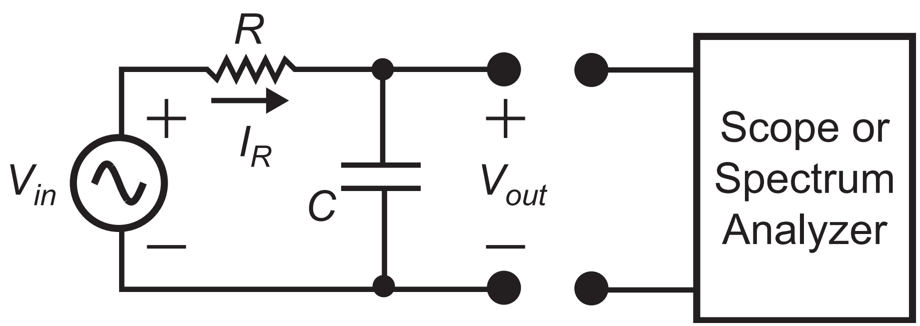
- **Derive impact of differentiation**

$$\frac{d}{dt}x(t) = \frac{d}{dt}\int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df$$

$$= \int_{-\infty}^{\infty} j2\pi f X(f) e^{j2\pi f t} df$$

$$\frac{d}{dt}x(t) \Leftrightarrow j2\pi f X(f)$$

Derivation of RC Filter Response



$$\frac{V_{in}(t) - V_{out}(t)}{R} = C\frac{dV_{out}(t)}{dt}$$

- **Apply FT to above differential equation**

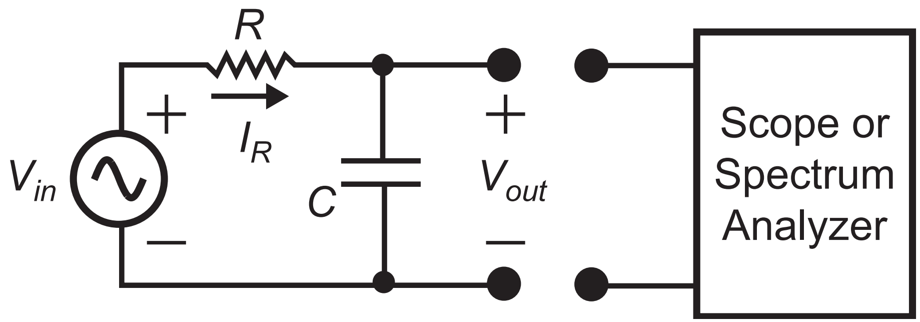
$$\frac{V_{in}(f) - V_{out}(f)}{R} = Cj2\pi f V_{out}(f)$$

$$\Rightarrow \frac{V_{in}(f)}{R} = \left(Cj2\pi f + \frac{1}{R}\right)V_{out}(f)$$

- **Filter frequency response is then calculated as**

$$H(f) = \frac{V_{out}(f)}{V_{in}(f)} = \frac{1}{1 + RCj2\pi f}$$

Magnitude of RC Filter Response



$$H(f) = \frac{1}{1 + RCj2\pi f}$$

- **Define cutoff frequency of filter**

$$f_c = \frac{1}{2\pi RC} \Rightarrow H(f) = \frac{1}{1 + jf/f_c}$$

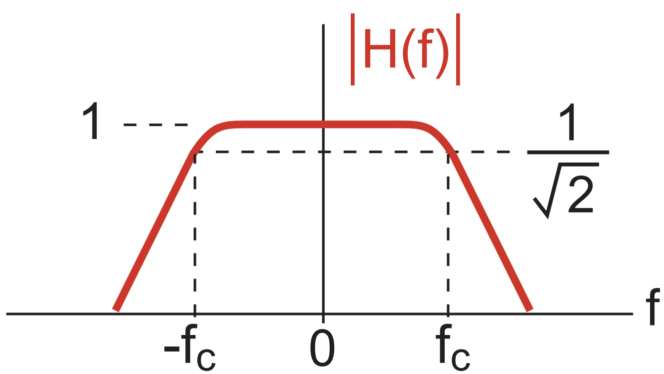
- **Magnitude of response:**

$$|H(f)| = \frac{1}{\sqrt{1 + (f/f_c)^2}}$$

$$f = f_c \Rightarrow |H(f)| = \frac{1}{\sqrt{2}}$$

$$f \ll f_c \Rightarrow |H(f)| \approx 1$$

$$f \gg f_c \Rightarrow |H(f)| \approx \frac{f_c}{f}$$



Summary

- **Coherent modulation requires synchronized local oscillators at transmitter and receiver**
  - Impact of phase offset is to change baseband *amplitude*
  - Impact of frequency offset is *fading* (small offset) or catastrophic *corruption* (large offset) of baseband signal
- **I/Q modulation allows twice the amount of information to be sent compared to basic AM**
  - Impact of phase offset is to swap I/Q
  - Impact of frequency offset is I/Q swapping (small offset) or catastrophic corruption (large offset) of received signal
- **RC networks provide *continuous-time* filtering**
- **Upcoming lectures**
  - Examine another non-ideality: noise
  - Lay groundwork for *digital* modulation and the concept of *information*