Analysis and Design of Analog Integrated Circuits
Lecture 12

Feedback

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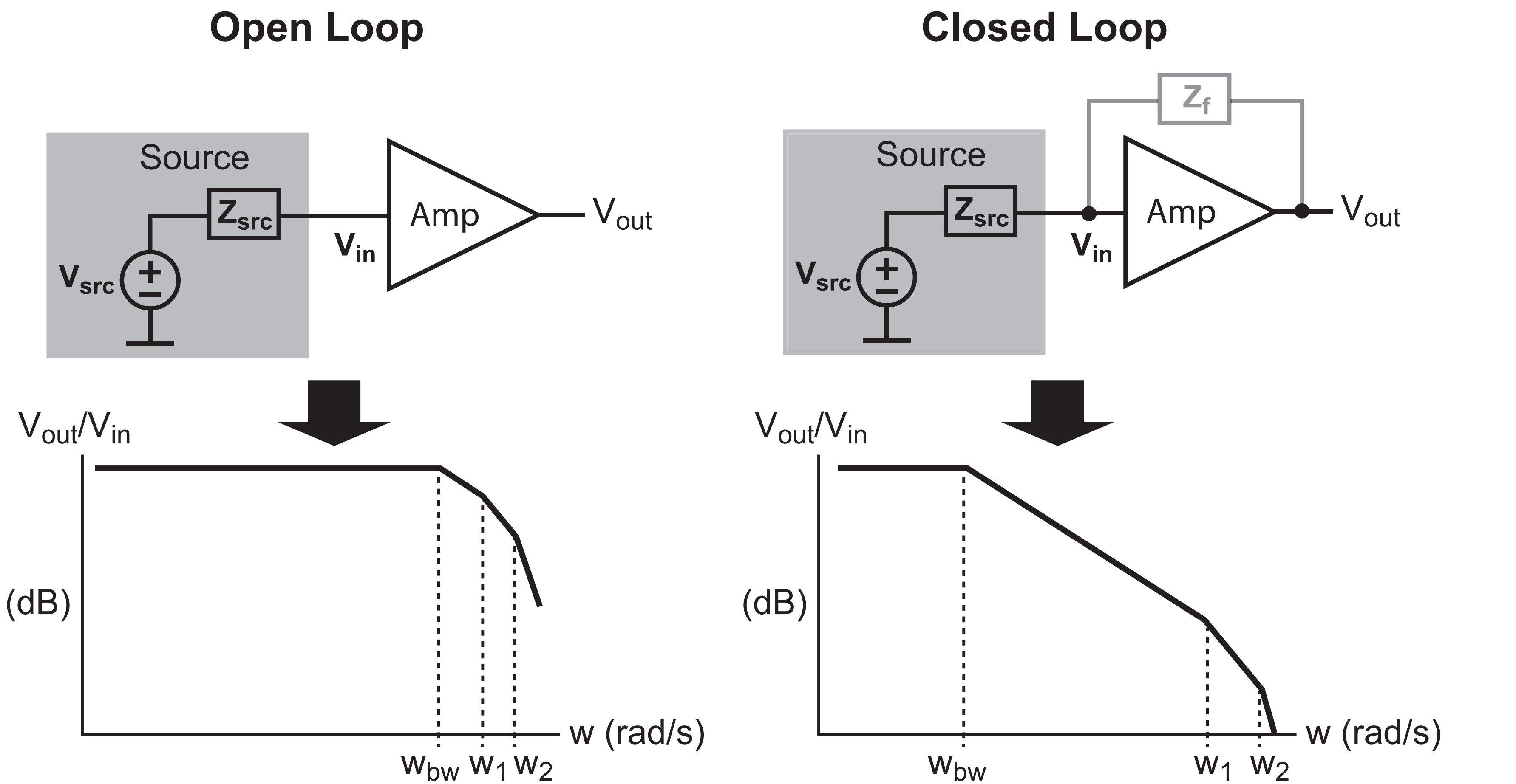
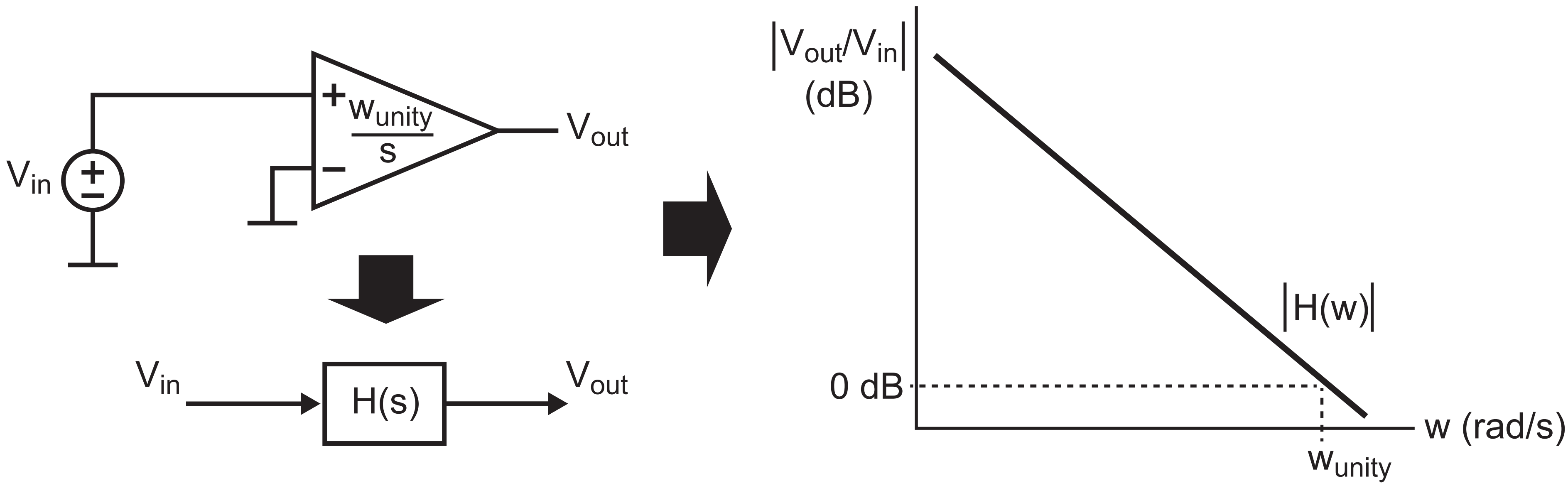
Open Loop Versus Closed Loop Amplifier Topologies



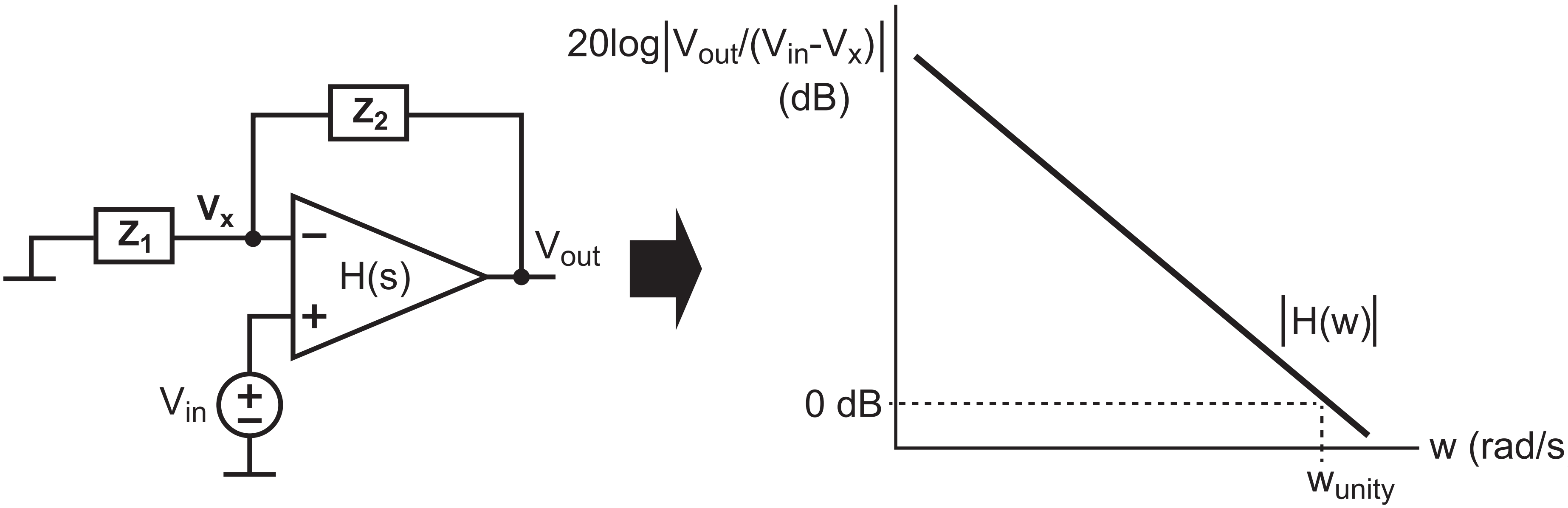
- Open loop – want all bandwidth limiting poles to be as high in frequency as possible
- Closed loop – want one pole to be dominant and all other parasitic poles to be as high in frequency as possible

Consider an Open Loop Integrator



- **Parameterize integrator in terms of its unity gain frequency, w_{unity} rad/s**
  - **Define $H(s) = w_{unity}/s$**
  - **Note that $|H(w_{unity})| = 1$**

Now Surround the Integrator with a Feedback Path

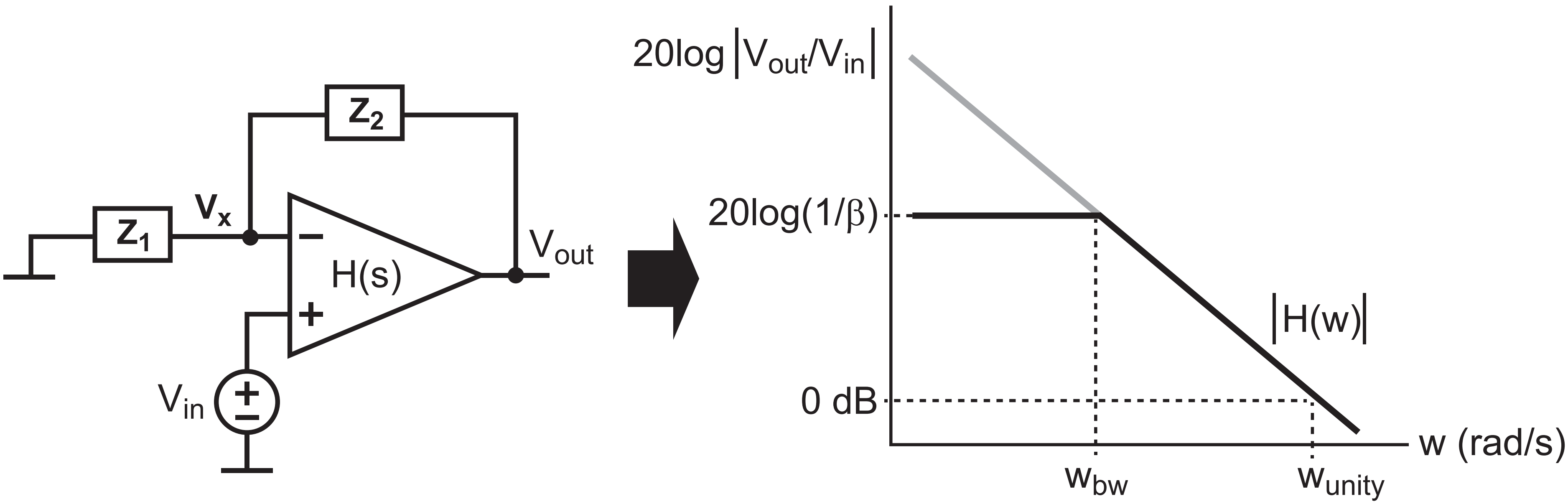


- **The *feedforward* path is *H(s) = w_{unity}/s***
- **The *feedback* path is formed by *Z_1* and *Z_2***
- **Derivation of *closed loop* transfer function:**

$$\frac{V_{out}-V_x}{Z_2} = \frac{V_x - 0}{Z_1} \quad \text{and} \quad V_{out} = H(s)(V_{in} - V_x)$$

$$\Rightarrow \quad \frac{V_{out}}{V_{in}} = \frac{H(s)}{1 + Z_1/(Z_1+Z_2)H(s)}$$

Observations of Impact of Feedback



- **Define $\beta = Z_1/(Z_1+Z_2)$ and rewrite transfer function as:**

$$\frac{V_{out}}{V_{in}} = \frac{H(s)}{1 + Z_1/(Z_1+Z_2)H(s)} = \frac{H(s)}{1+\beta \cdot H(s)}$$

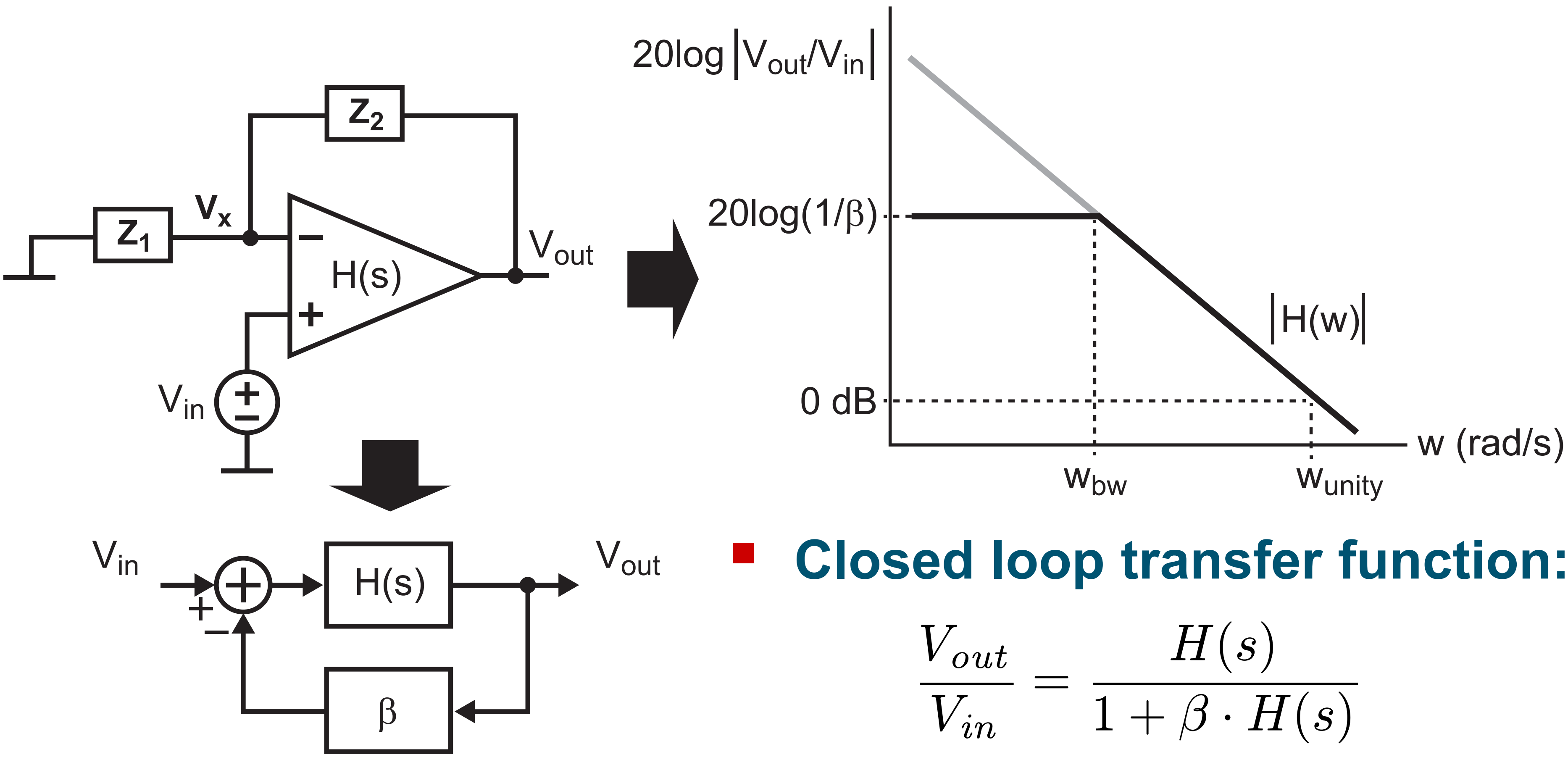
- **At low frequencies:** **At high frequencies:**

$$\left|\frac{V_{out}}{V_{in}}\right|_{s\rightarrow 0} = \frac{\infty}{1+\beta\cdot\infty} = \frac{1}{\beta}$$

$$\left|\frac{V_{out}}{V_{in}}\right|_{s\rightarrow \infty} = |H(w)|$$

$$(\text{since } |\beta \cdot H(w)| \ll 1)$$

General View of Feedback



- **Closed loop transfer function:**

$$\frac{V_{out}}{V_{in}} = \frac{H(s)}{1+\beta\cdot H(s)}$$

  - **This is called Black's formula**

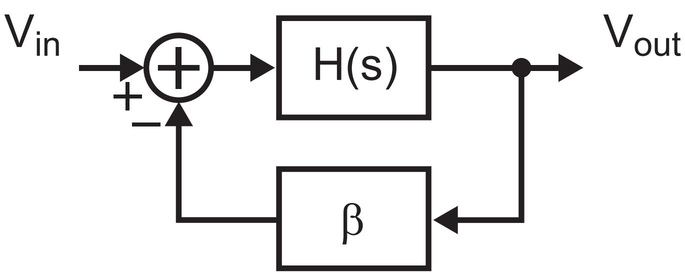
- **At low frequencies:**

$$\left|\frac{V_{out}}{V_{in}}\right|_{s\rightarrow 0} = \frac{1}{\beta}$$

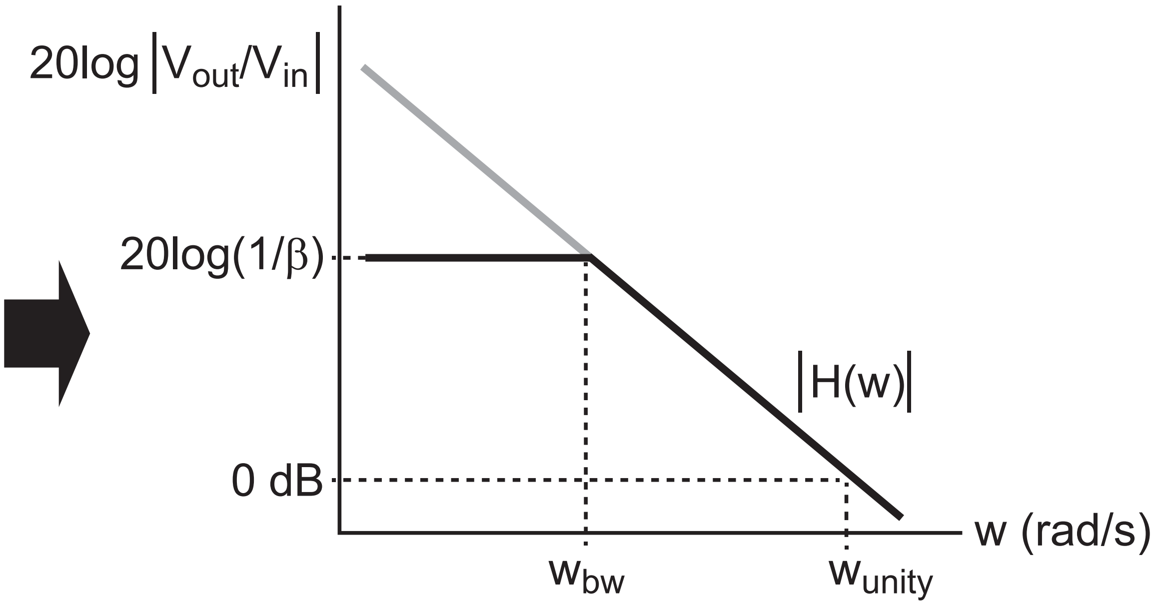
At high frequencies:

$$\left|\frac{V_{out}}{V_{in}}\right|_{s\rightarrow \infty} = |H(w)|$$

General Observations of Feedback

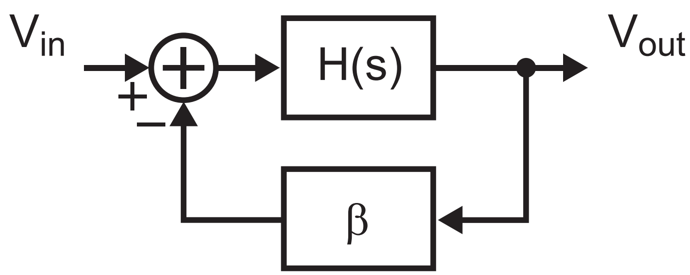


$$\frac{V_{out}}{V_{in}} = \frac{H(s)}{1 + \beta \cdot H(s)}$$

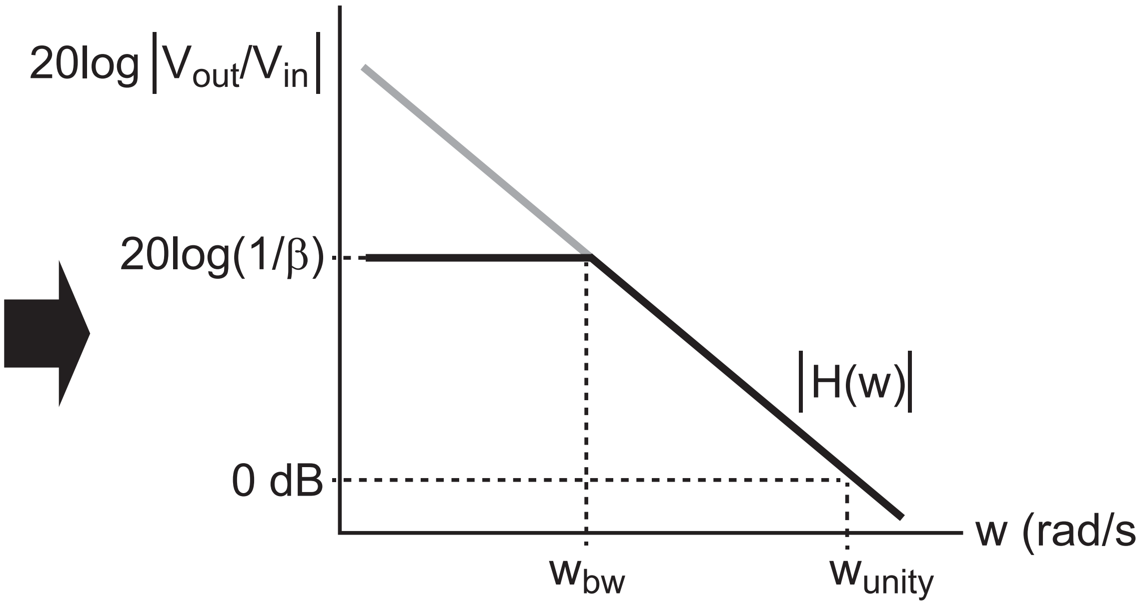


- **The *feedback path* sets the closed loop gain at low frequencies**
  - **Assumes the open loop gain is large at low frequencies**
  - **Implies that accurate *closed loop* gain can be achieved at low frequencies despite variations in *open loop* gain**
- **The *feedback path* also influences the closed loop bandwidth**

Gain Bandwidth Product for Closed Loop Systems



$$\frac{V_{out}}{V_{in}} = \frac{H(s)}{1+\beta \cdot H(s)}$$

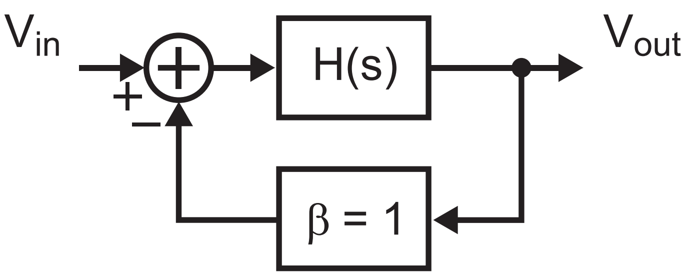


- **The low frequency gain is:** $\left|\frac{V_{out}}{V_{in}}\right|_{s\rightarrow 0} = \frac{1}{\beta}$
- **The bandwidth roughly corresponds to:** $|H(w_{bw})| \approx \frac{1}{\beta}$

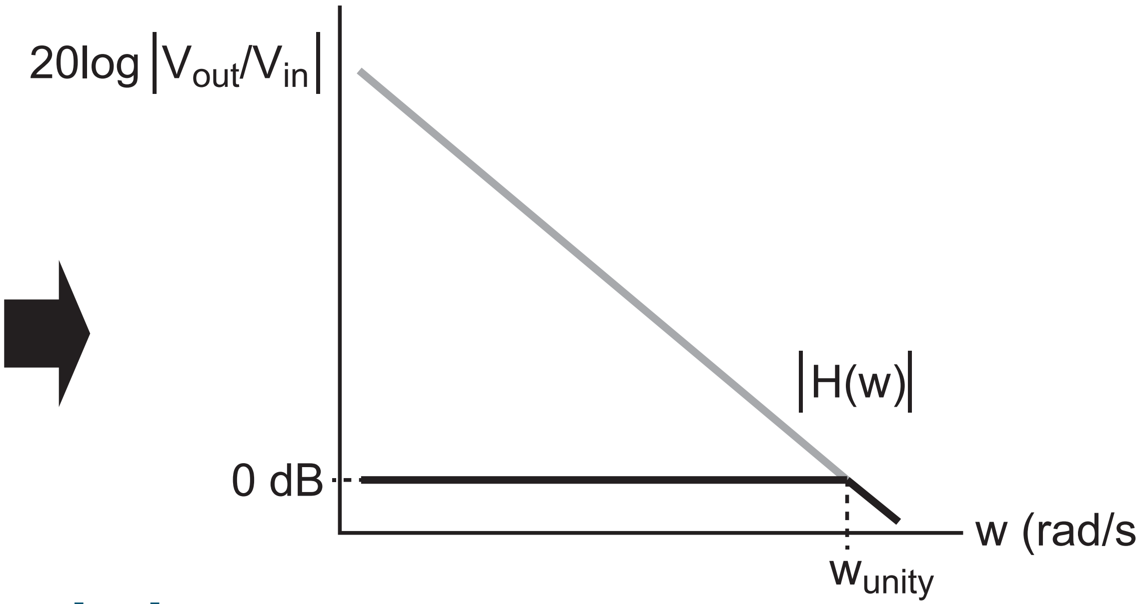
$$\text{For } H(s) = \frac{w_{unity}}{s} \quad \Rightarrow \left|\frac{w_{unity}}{w_{bw}}\right| \approx \frac{1}{\beta} \Rightarrow w_{unity} = \frac{1}{\beta} \cdot w_{bw}$$

Closed loop systems exhibit constant gain-bandwidth product set by the unity gain frequency of the open loop amplifier

Example: Unity Gain Amplifier



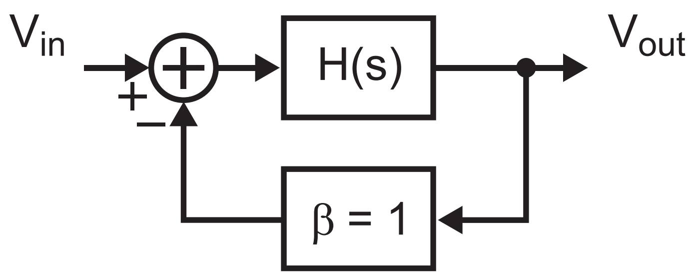
$$\frac{V_{out}}{V_{in}} = \frac{H(s)}{1+H(s)}$$



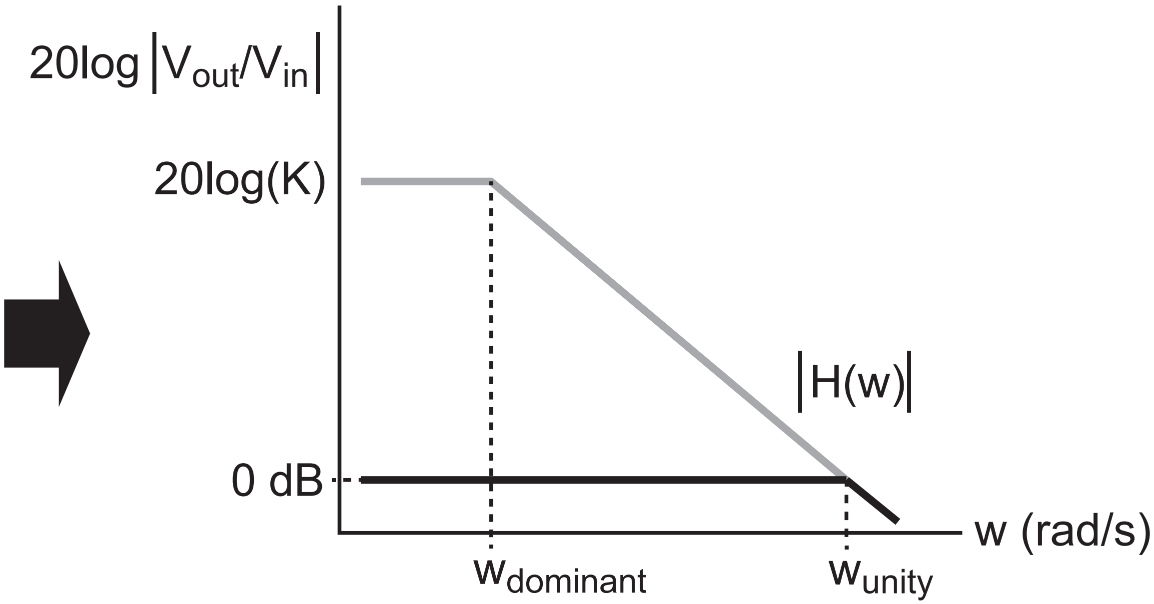
- **The low frequency gain is:** $$\left|\frac{V_{out}}{V_{in}}\right|_{s\to 0} = 1$$
- **The bandwidth roughly corresponds to:** $|H(w_{bw})| \approx w_{unity}$
- **Gain bandwidth product is *w*** **w_{unity}:** $1 \cdot w_{unity} = w_{unity}$

Unity gain closed loop amplifiers maximize the closed loop bandwidth assuming closed loop gain ≥ 1

Issue: Open Loop Amplifiers have Finite DC Gain



$$\frac{V_{out}}{V_{in}} = \frac{H(s)}{1+H(s)}$$



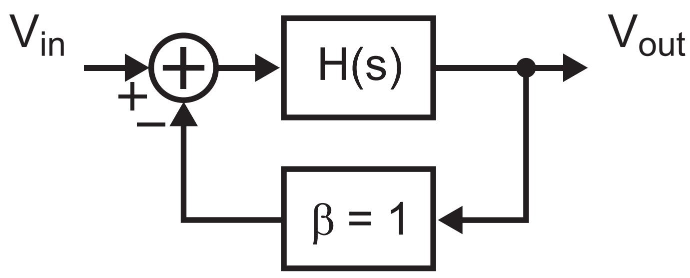
- **Let us now model *H(s)* as:**

$$H(s) = \frac{K}{1+s/w_{dominant}}$$

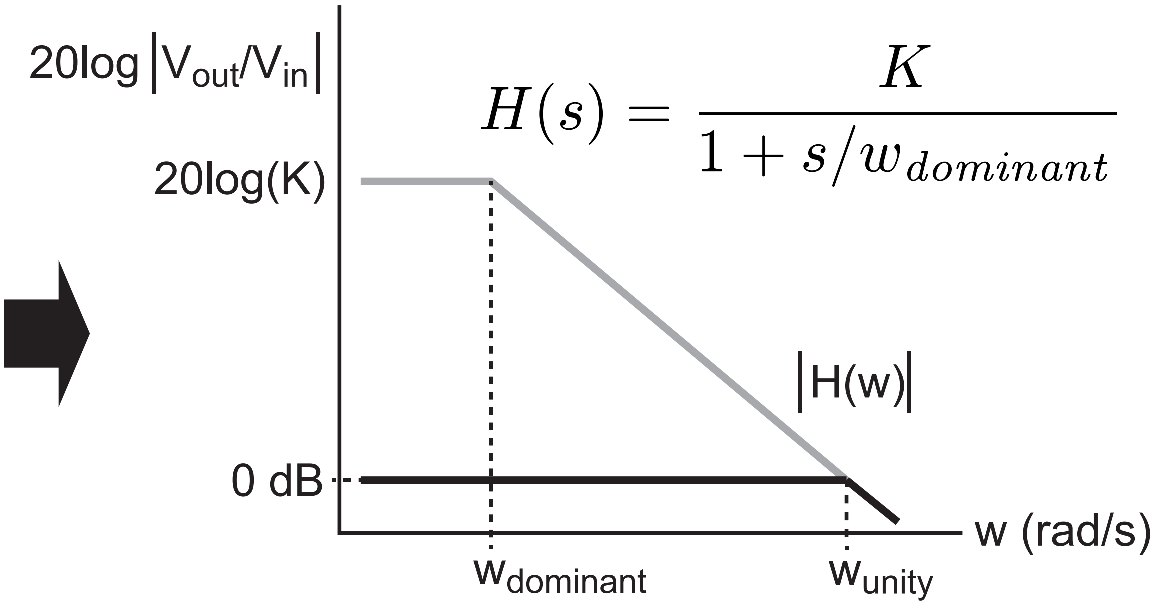
- **To first order, the closed loop bandwidth and gain are relatively unchanged**

What is the impact of having finite, open loop, DC gain?

Further Examination of Finite, Open Loop, DC Gain



$$\frac{V_{out}}{V_{in}} = \frac{H(s)}{1+H(s)}$$



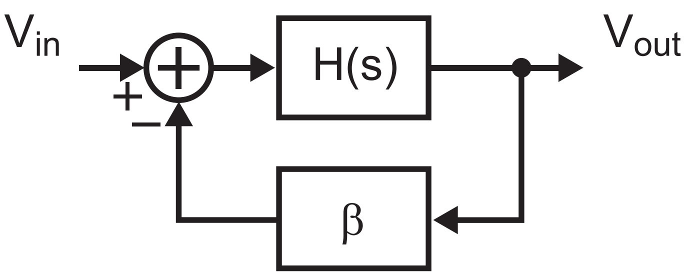
$$H(s) = \frac{K}{1+s/w_{dominant}}$$

- For unity gain configuration of closed loop amplifier:

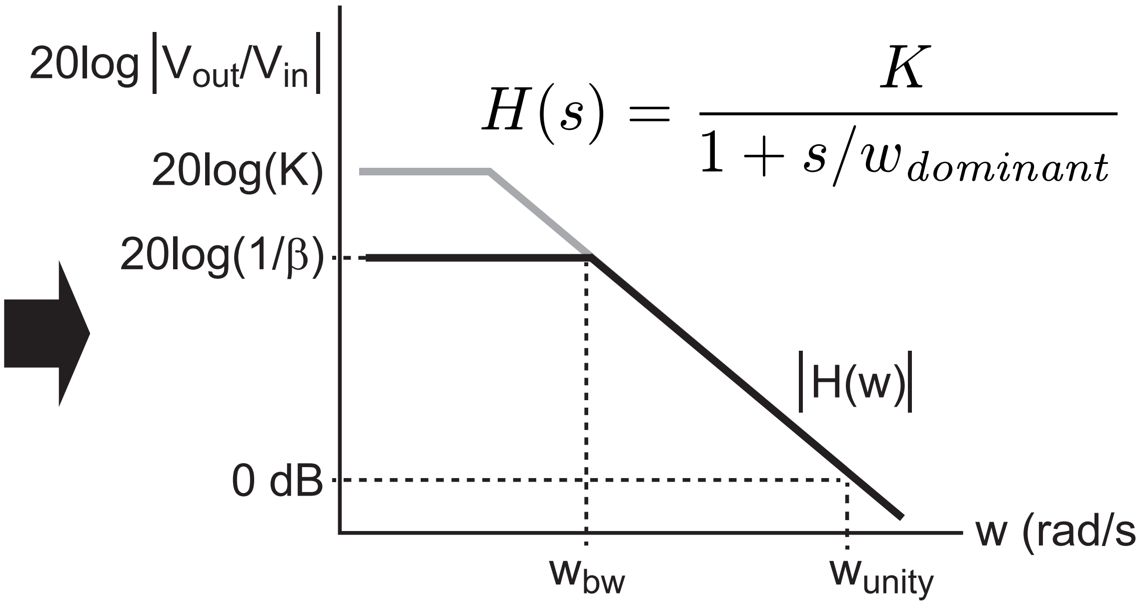
$$\left|\frac{V_{out}}{V_{in}}\right|_{s\to 0} = \left.\frac{H(s)}{1+H(s)}\right|_{s\to 0} = \frac{K}{1+K} = \frac{1}{1+1/K}$$

- We see that finite open loop DC gain leads to a slight reduction of the closed loop DC gain
  - We want *K* >> 1 for the unity gain closed loop amplifier

More General View of Finite, Open Loop, DC Gain



$$\frac{V_{out}}{V_{in}} = \frac{H(s)}{1+\beta \cdot H(s)}$$



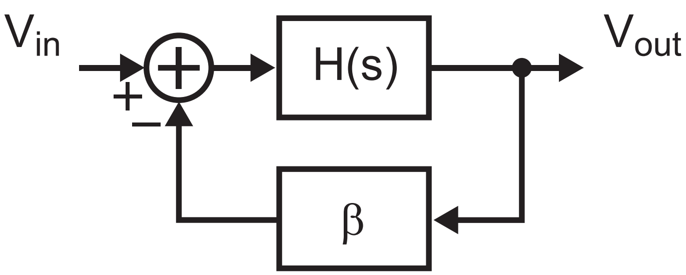
$$H(s) = \frac{K}{1+s/w_{dominant}}$$

- **For general configuration of closed loop amplifier:**

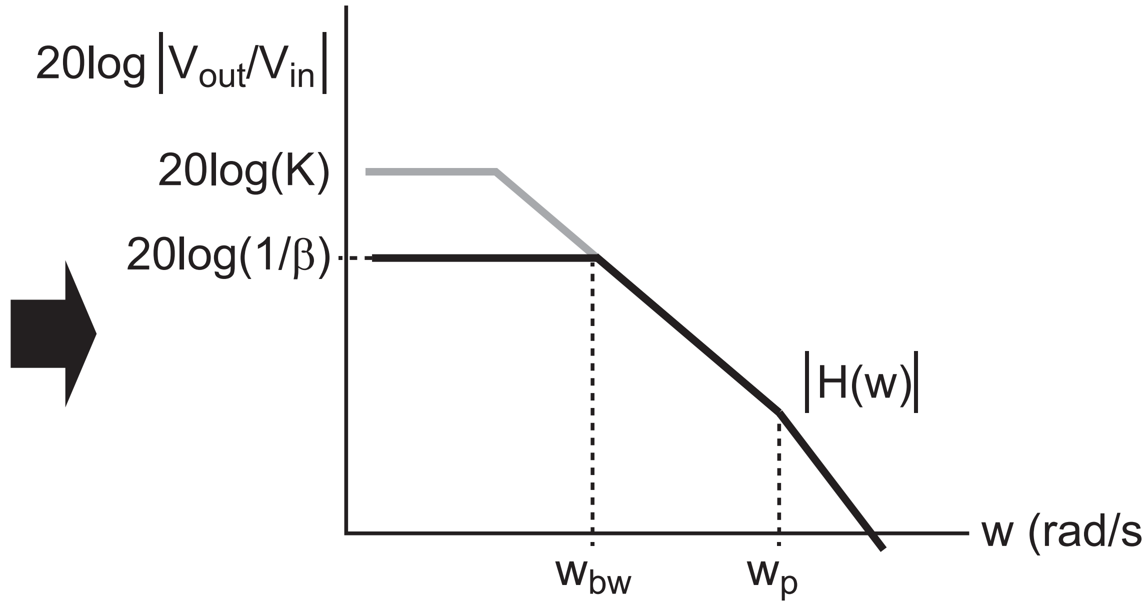
$$\left|\frac{V_{out}}{V_{in}}\right|_{s\rightarrow 0} = \frac{K}{1+\beta \cdot K} = \left(\frac{1}{\beta}\right)\frac{1}{1+(1/\beta)/K}$$

- **Finite open loop DC gain still leads to reduction of closed loop DC gain**
  - **We want *K >> 1/β* in this case**
  - **We will see implications of this issue later in the class**

The Issue of Parasitic Open Loop Poles



$$\frac{V_{out}}{V_{in}} = \frac{H(s)}{1+\beta\cdot H(s)}$$

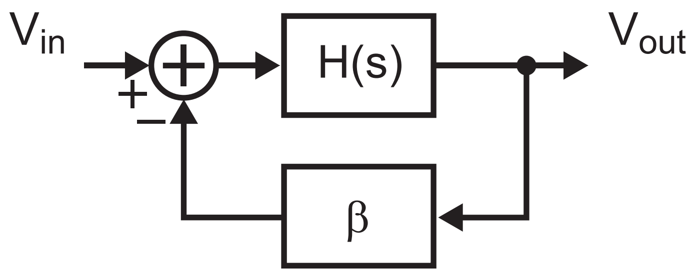


- **Practical amplifiers have non-dominant poles, too:**

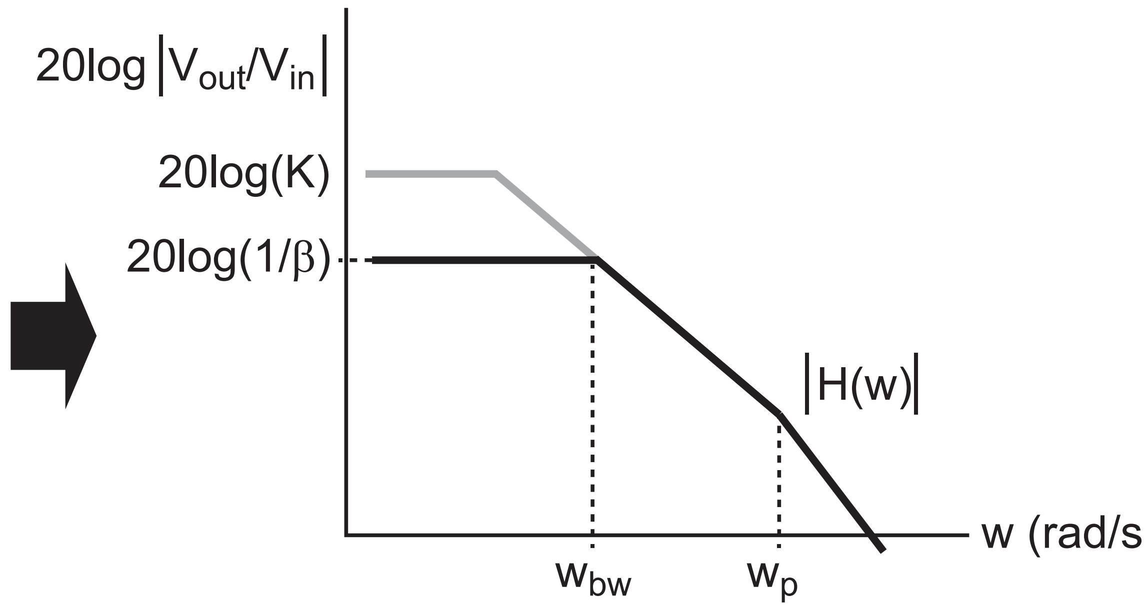
$$H(s) = \left(\frac{K}{1+s/w_{dominant}}\right)\left(\frac{1}{1+s/w_p}\right)$$

  - **Of course, there can be multiple parasitic poles and also zeros**
- **A key issue of such parasitic poles is their influence on the *stability* of the closed loop amplifier**

Key Tool for Assessing Stability: Open Loop Response



$$\frac{V_{out}}{V_{in}} = \frac{H(s)}{1+\beta\cdot H(s)}$$



- **We define the open loop response, *A(s)*, as:**

$$A(s) = \beta\cdot H(s)$$

- **Note that the unity gain frequency, w_0, of *A(w)* is approximately the same as the closed loop bandwidth, w_{bw}**

$$|A(w_0)| = 1 \;\Rightarrow\; \beta\cdot|H(w_0)| = 1 \;\Rightarrow\; |H(w_0)| = 1/\beta$$

  – **Looking at the plot above, we can see that the intersection of |H(w)| and 1/β corresponds to the closed loop bandwidth, w_{bw}**

Stability Analysis Based on Phase Margin of A(w)

- ***Phase margin*** **is a key metric when examining the stability of a system**
  - **Phase margin is defined as 180° + phase{$A(w_0)$}**
    - **w_0 corresponds to the unity gain frequency of the open loop response (i.e., $|A(w_0)| = 1$)**
    - **w_0 is *approximately* the same as the closed loop bandwidth, w_{bw}**
  - **Phase margin must be greater than 0 degrees for the closed loop system to be stable**
    - **Typically want phase margin to be greater than 45°**

- **Key skill: you must be able to plot Bode plots in both magnitude and phase!**

Review of Bode Plot Basics

- **Example:**

$$A(w) = \frac{1 + jw/w_z}{(1 + jw/w_{p1})(1 + jw/w_{p2})}$$

  - **Log of magnitude (dB):** $20\log|A(w)|$

$$= 20\log|1 + jw/w_z| - 20\log|1 + jw/w_{p1}| - 20\log|1 + jw/w_{p2}|$$

    - Taking the log allows the poles and zeros to be plotted separately and then added together

  - **Phase:** $\angle A(w)$

$$= \angle(1 + jw/w_z) - \angle(1 + jw/w_{p1}) - \angle(1 + jw/w_{p2})$$

    - Phase of poles and zeros can also be plotted separately and then added together

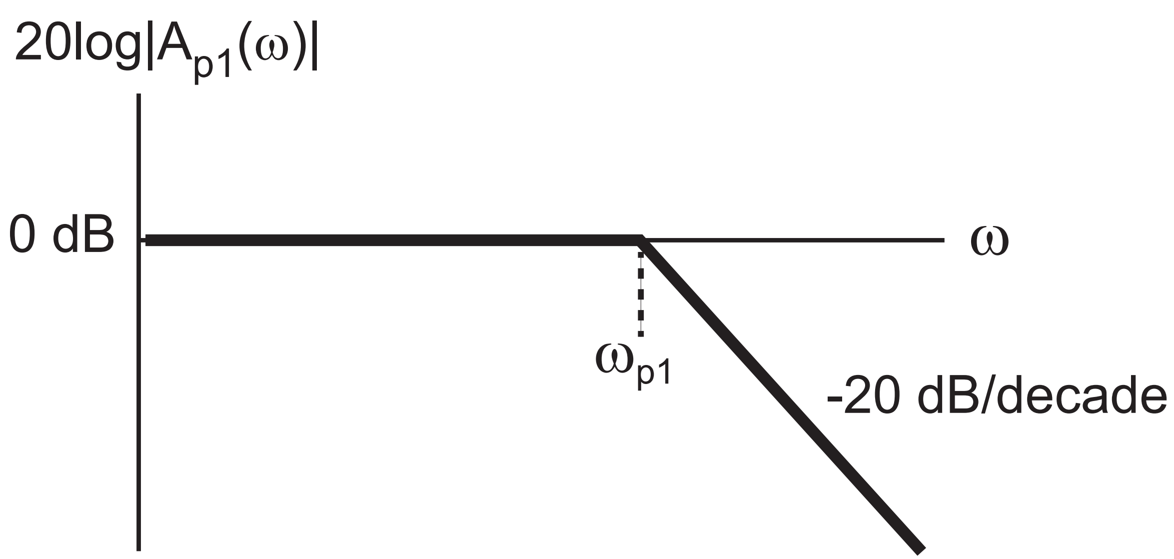
Review: Plotting the Magnitude of Poles

- **Plot the magnitude response of pole w_{p1}**

$$20\log|A_{p1}(w)| = 20\log\left|\frac{1}{1+jw/w_{p1}}\right| = -20\log|1+jw/w_{p1}|$$

  - **For $w \ll w_{p1}$:** $20\log|A_{p1}(w)| \approx -20\log|1| = 0$

  - **For $w \gg w_{p1}$:** $20\log|A_{p1}(w)| \approx -20\log|w/w_{p1}|$

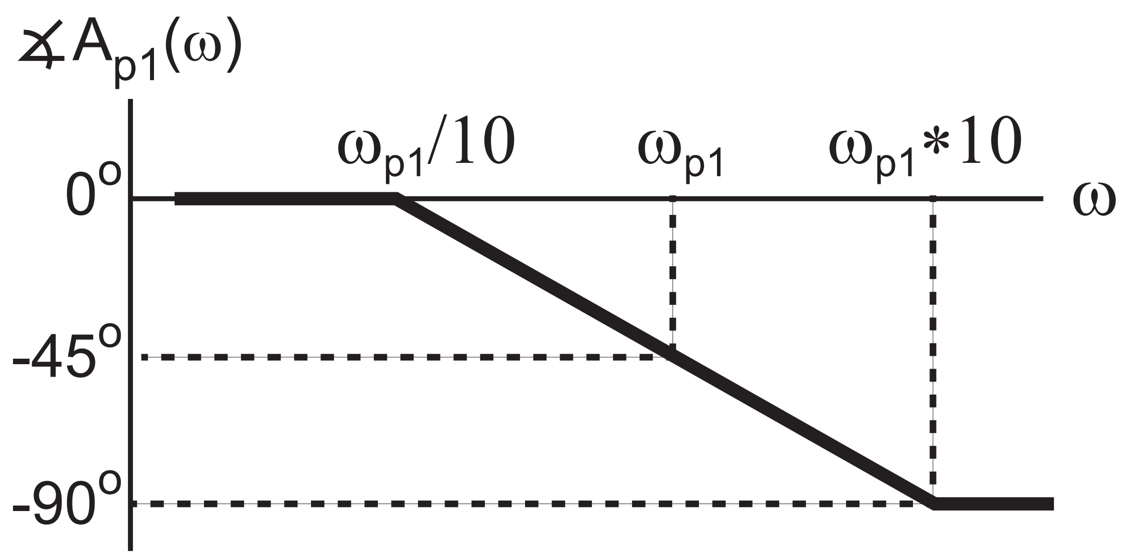


Plotting the Phase of Poles

- **Plot the phase response of pole w_{p1}**

$$\angle A_{p1}(w) = -\angle(1 + jw/w_{p1}) = -\arctan(w/w_{p1})$$

  - **For $w << w_{p1}$:** $\angle A_{p1}(w) \approx -\arctan(0) = 0^\circ$
  - **For $w = w_{p1}$:** $\angle A_{p1}(w) \approx -\arctan(1) = -45^\circ$
  - **For $w >> w_{p1}$:** $\angle A_{p1}(w) \approx -\arctan(\infty) = -90^\circ$

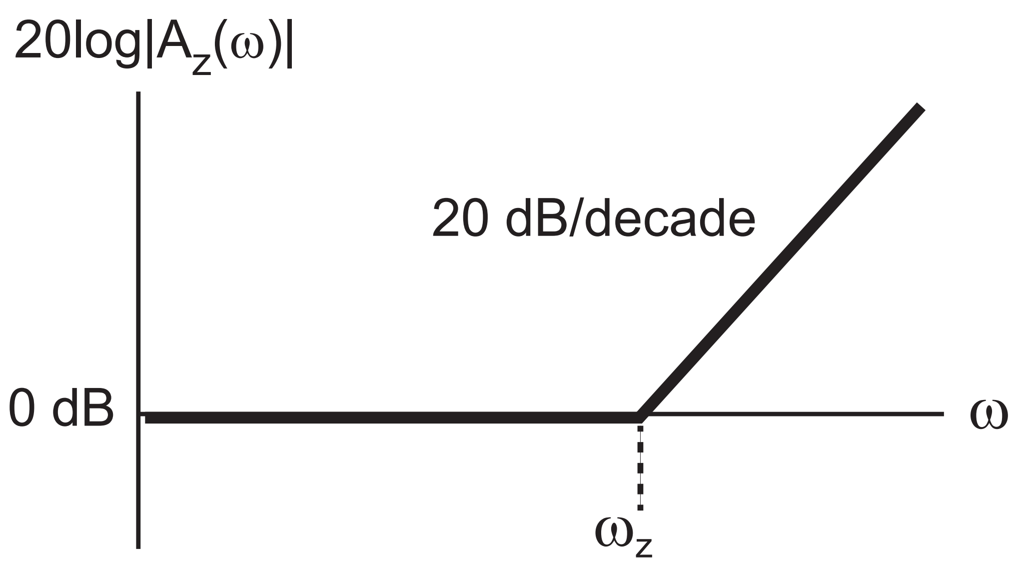


Review: Plotting the Magnitude of Zeros

- **Plot the magnitude response of zero w_z**

$$20\log|A_z(w)| = 20\log|1 + jw/w_z|$$

  - **For $w \ll w_z$:** $$20\log|A_z(w)| \approx 20\log|1| = 0$$
  - **For $w \gg w_z$:** $$20\log|A_z(w)| \approx 20\log|w/w_z|$$

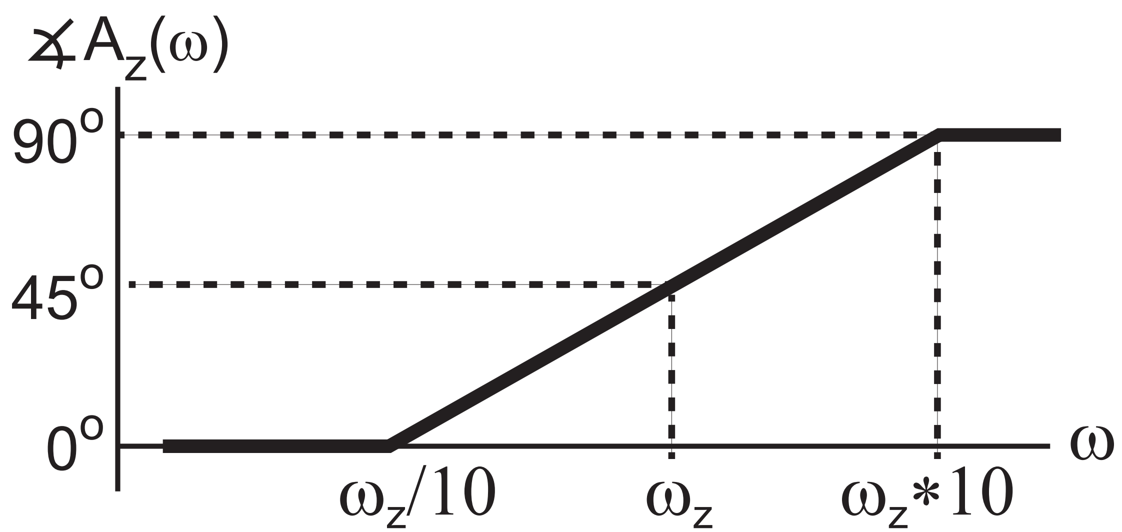


Plotting the Phase of Zeros

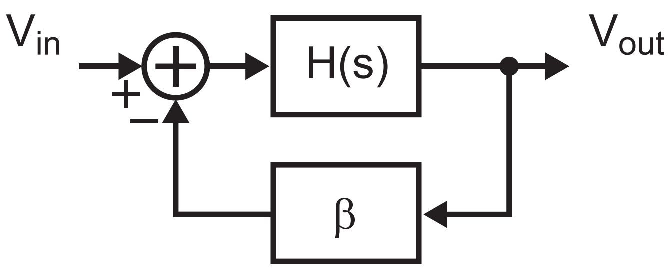
- **Plot the phase response of zero w_z**

$$\angle A_z(w) = \angle(1 + jw/w_z) = \arctan(w/w_z)$$

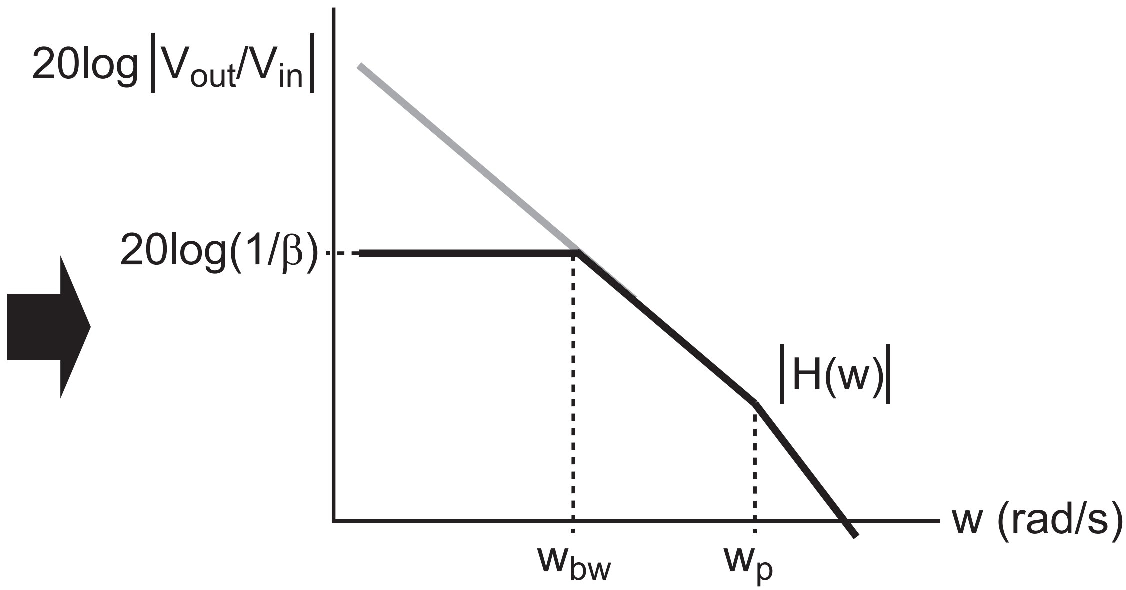
  - **For $w << w_z$:** $$\angle A_z(w) \approx \arctan(0) = 0^\circ$$
  - **For $w = w_z$:** $$\angle A_z(w) \approx \arctan(1) = 45^\circ$$
  - **For $w >> w_z$:** $$\angle A_z(w) \approx \arctan(\infty) = 90^\circ$$



Example of Closed Loop Stability Evaluation



$$\frac{V_{out}}{V_{in}} = \frac{H(s)}{1 + \beta \cdot H(s)}$$



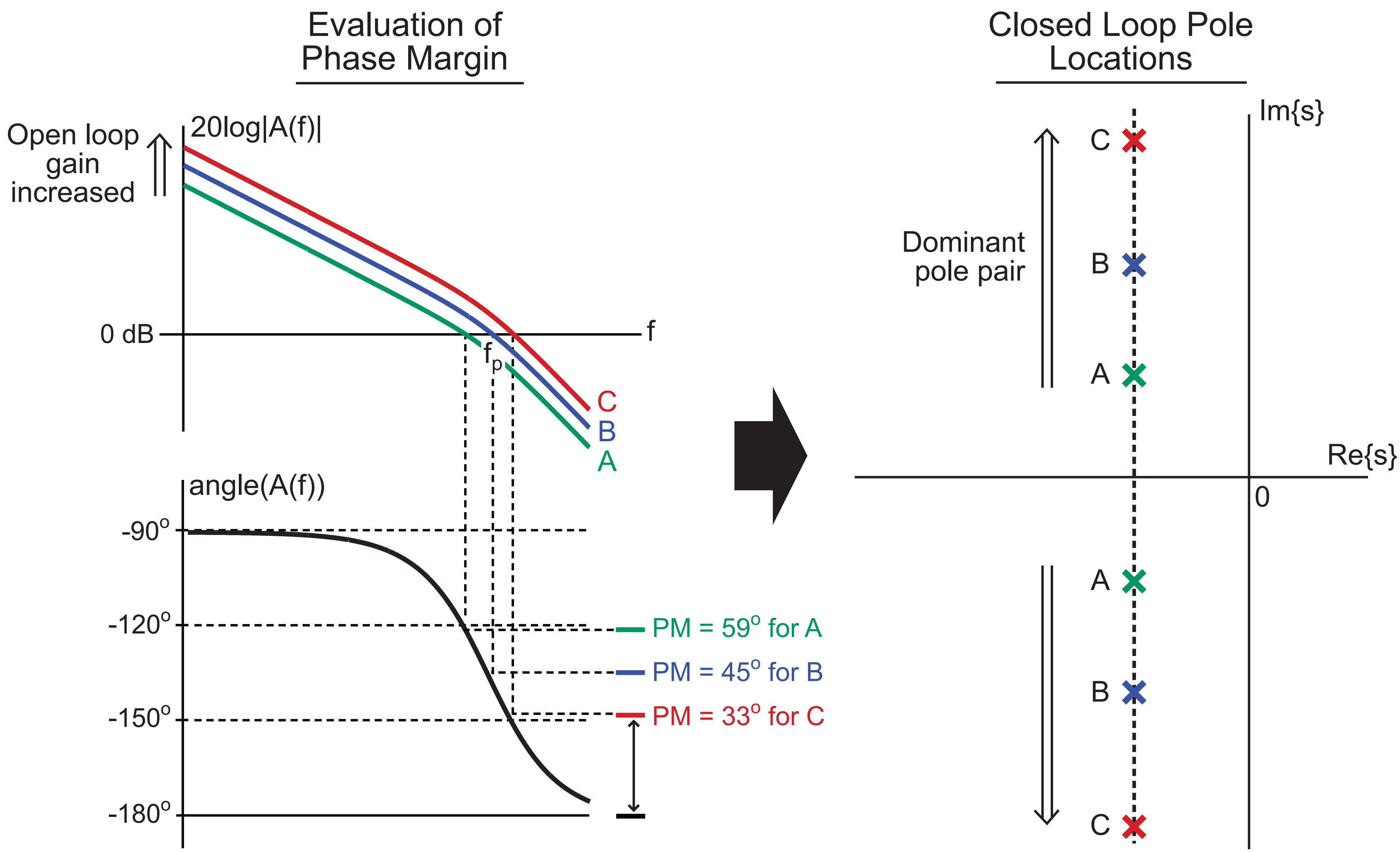
- **Consider the case where:**

$$H(s) = \left(\frac{K}{s}\right)\left(\frac{1}{1 + s/w_p}\right)$$

  - **This implies that:**

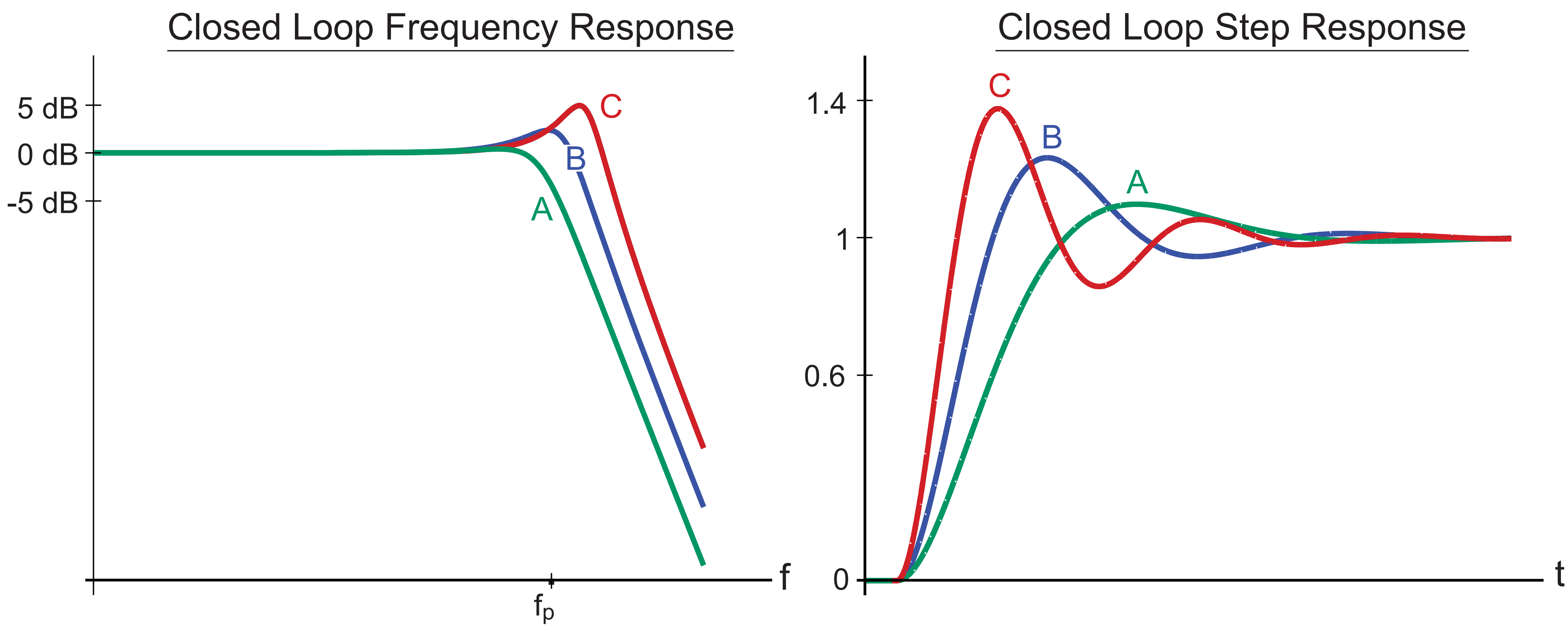
$$A(s) = \beta\left(\frac{K}{s}\right)\left(\frac{1}{1 + s/w_p}\right)$$

Phase Margin Versus Open Loop Gain



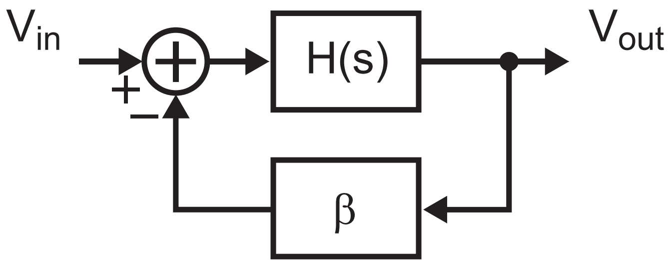
- Note the closed loop pole locations versus open loop gain
  - Is the closed loop system unstable for any case above?

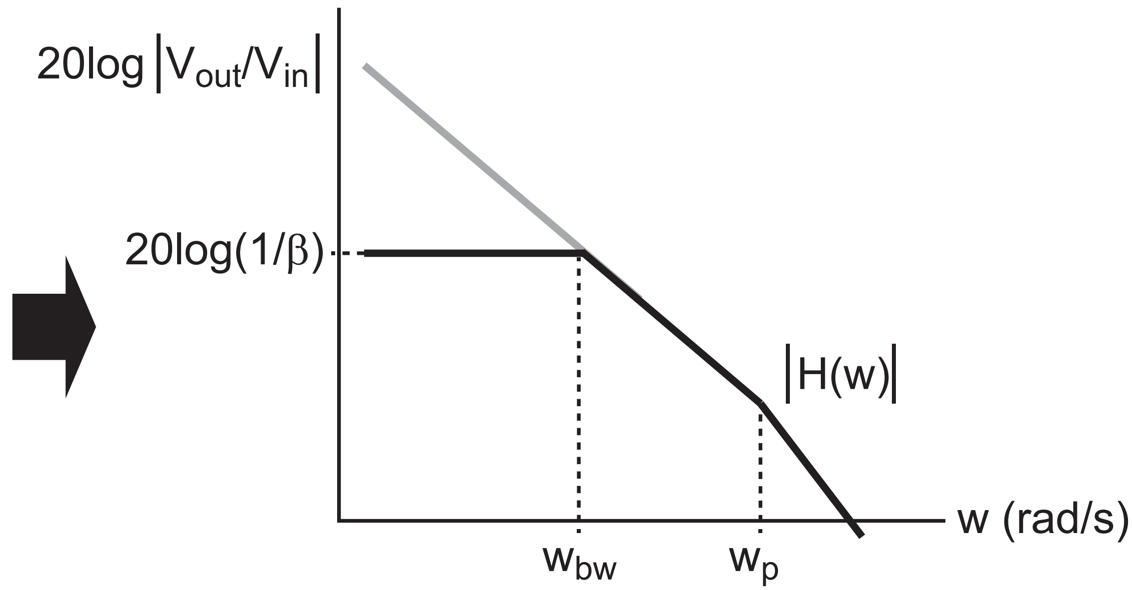
Corresponding Closed Loop Behavior



- **Frequency response sees more peaking with higher open loop gain**
  - **How does this relate to the movement of the closed loop pole locations?**
- **Step response see more ringing with higher open loop gain**
  - **How does this relate to the closed loop frequency response?**

Some Key Observations



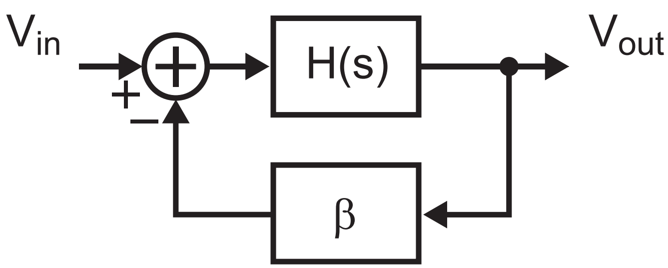


$$\frac{V_{out}}{V_{in}} = \frac{H(s)}{1+\beta \cdot H(s)}$$

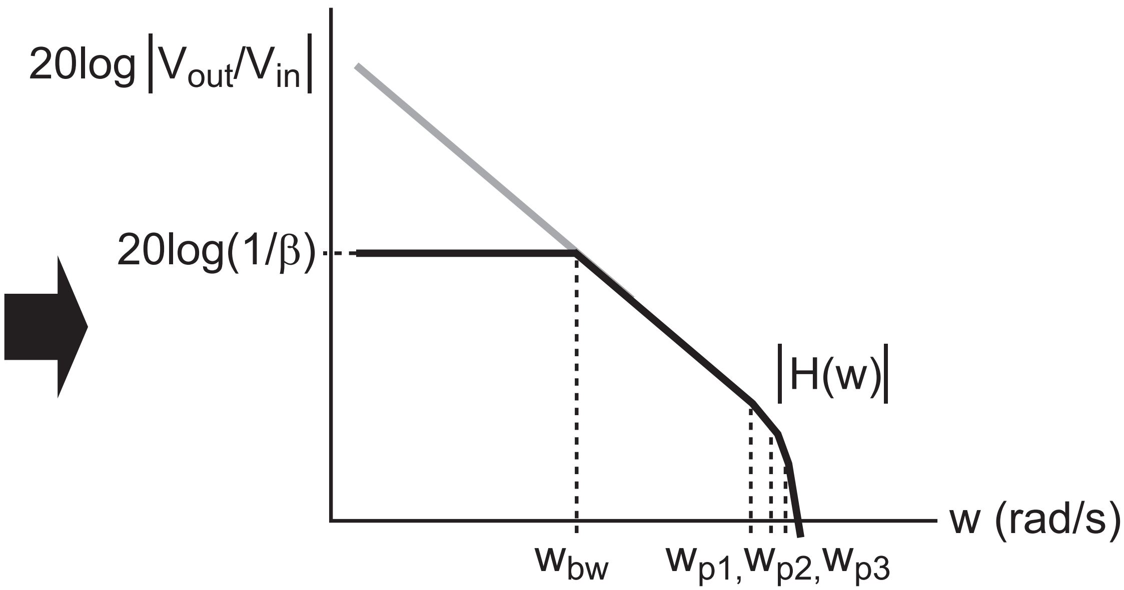
$$A(s) = \beta \cdot H(s)$$

- **We have seen that increasing the open loop gain of *A(w)* leads to *higher* closed loop bandwidth**
  - **How is this consistent with the statement that increasing closed loop gain leads to *lower* closed loop bandwidth?**
- **As an exercise, consider the impact of the following:**
  - **Keep β unchanged and increase the open loop gain of *H(w)***
  - **Keep *H(w)* unchanged and increase β**

Example 2 of Closed Loop Stability Evaluation



$$\frac{V_{out}}{V_{in}} = \frac{H(s)}{1 + \beta \cdot H(s)}$$



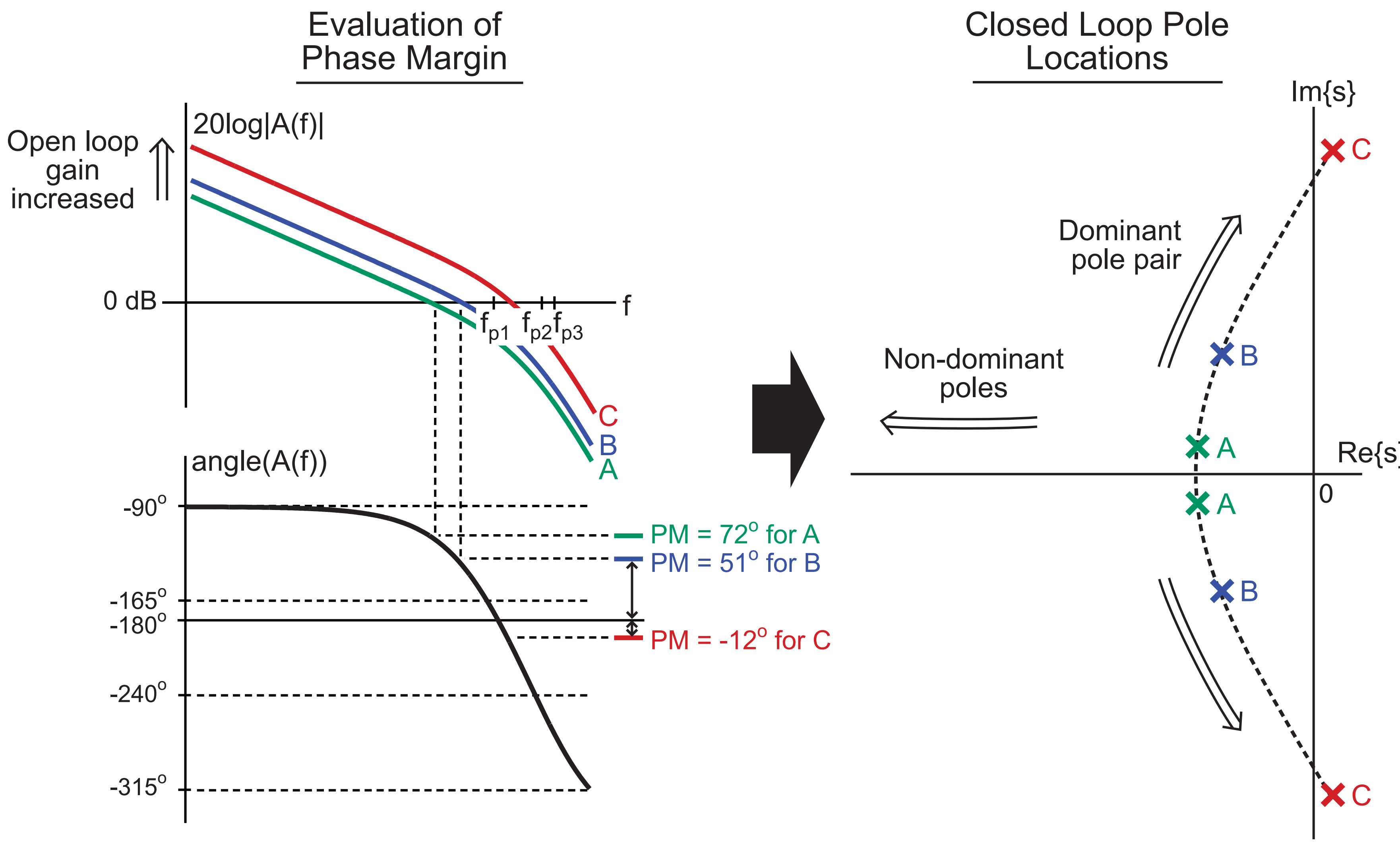
- **Consider the case where:**

$$H(s) = \left(\frac{K}{s}\right)\left(\frac{1}{1+s/w_{p1}}\frac{1}{1+s/w_{p2}}\frac{1}{1+s/w_{p3}}\right)$$

  - **This implies that:**

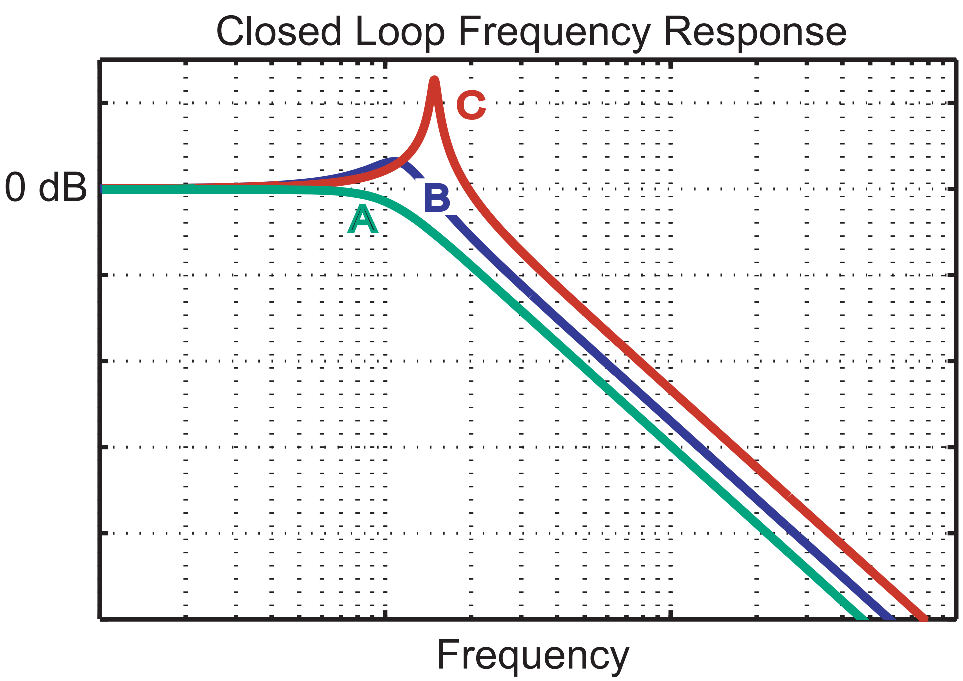
$$A(s) = \beta\left(\frac{K}{s}\right)\left(\frac{1}{1+s/w_{p1}}\frac{1}{1+s/w_{p2}}\frac{1}{1+s/w_{p3}}\right)$$

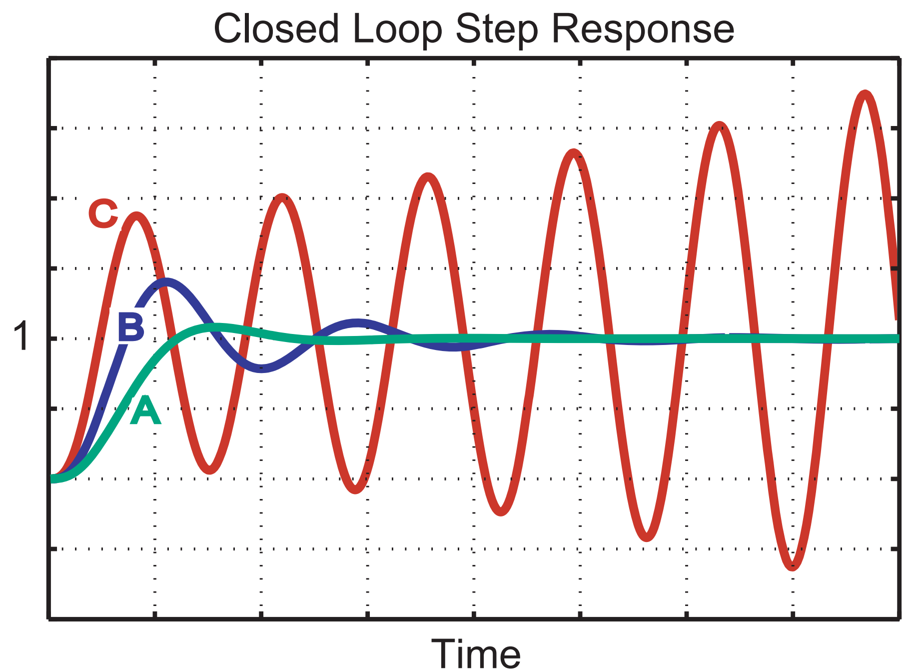
Phase Margin Versus Open Loop Gain



- **Note the closed loop pole locations versus open loop gain**
  - **Is the closed loop system unstable for any case above?**

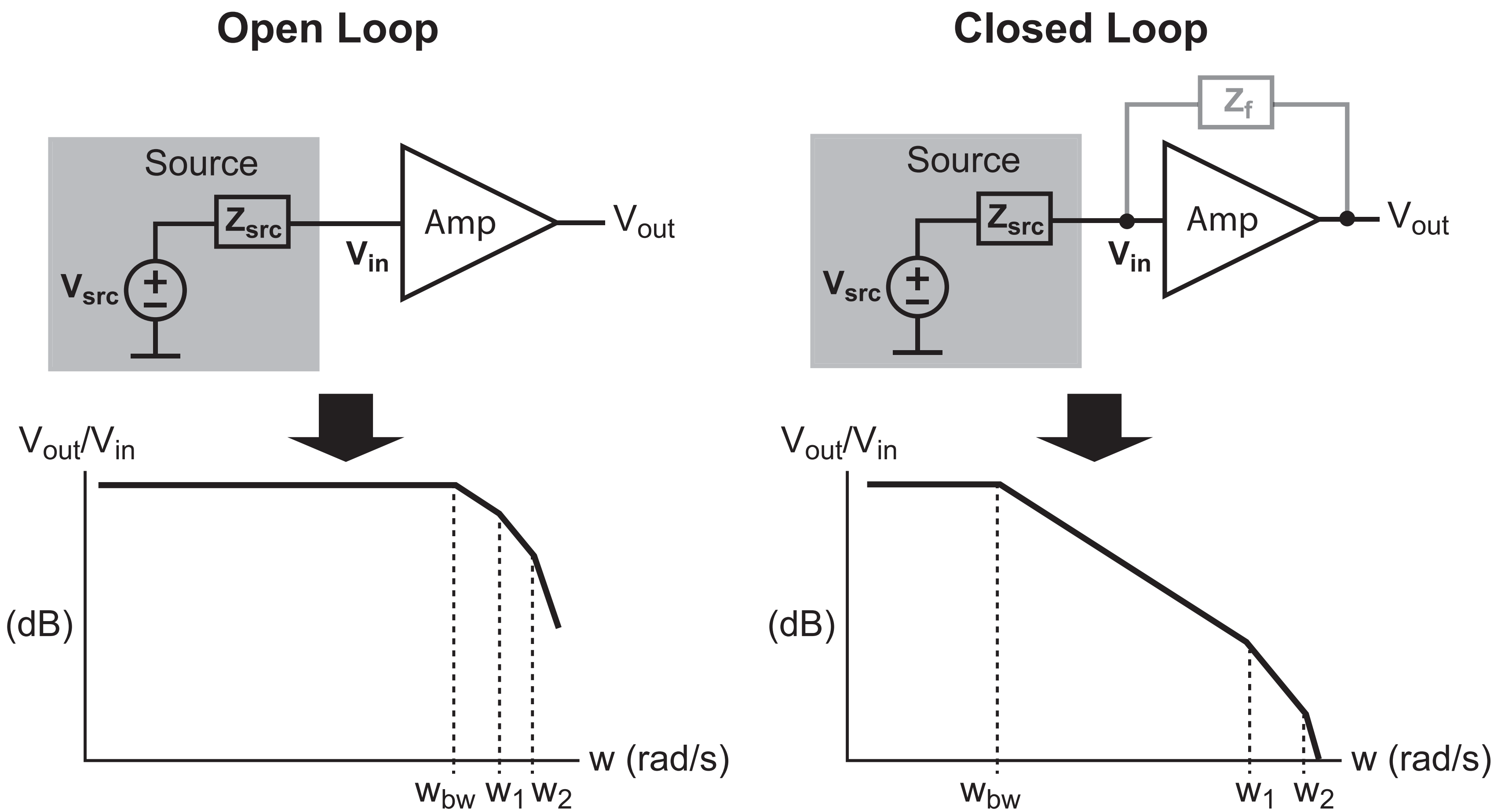
Corresponding Closed Loop Behavior





- **Frequency response again sees more peaking with higher open loop gain**
  - **How does this relate to the movement of the closed loop pole locations?**
- **Step response ringing grows for high open loop gain**
  - **How does this relate to the closed loop pole locations?**

Open Loop Versus Closed Loop Amplifier Topologies



- **Now that we understand the phase margin criterion, can you explain why amplifiers designed to be within a closed loop system should have one dominant pole that is much lower in frequency than the parasitic poles?**

Summary

- **Feedback systems offer the benefit of accurate gain at low frequencies**
  - **Assumes accurate feedback and high open loop DC gain**
  - **Gain-bandwidth product of the closed loop system equals w_{unity} of the open loop amplifier**
- **Accuracy of the closed loop DC gain is reduced with lower open loop DC gain**
  - **Want the open loop DC gain to be much higher than the desired closed loop DC gain for reasonable accuracy**
- **Stability of the closed loop system is often evaluated using the phase margin criterion**
  - **Examines the phase at unity gain frequency of the open loop response, $A(w_0) = \beta \cdot H(w_0)$, where $|A(w_0)| = 1$**
    - *w_0 is approximately the same as the closed loop bandwidth, w_{bw}*