## Short Course On Phase-Locked Loops and Their Applications Day 3, AM Lecture

#### **Advanced Analog Synthesizer Techniques**

Michael Perrott August 13, 2008

Copyright © 2008 by Michael H. Perrott All rights reserved.

#### **Outline**

- Frequency and phase modulation
  - Leveraging Fractional-N synthesizers for this task
- PLL filter compensation
- Sigma-Delta quantization noise cancellation
- Fast and accurate behavioral simulation

# **Constant Envelope Modulation**



- Popular for cell phones and cordless phones due to the reduced linearity requirements on the power amp
  - Allows a more efficient power amp design
    - Transmitter power is reduced

# Frequency Shift Keying



- Sends information encoded in instantaneous frequency
  - Can build simple transmitters and receivers
    - Pagers use this modulation method
- Issue want to obtain high spectral efficiency
  - Need to choose an appropriate transmit filter
  - **Need to choose an appropriate value of**  $\Delta f$

# A More Detailed Model



- The choice of ∆f is now parameterized by h and T<sub>d</sub>
  - h is called the modulation index, T<sub>d</sub> is symbol period

Note: phase modulation has *nonlinear* impact on I and Q!

#### **MSK Modulation**



Choose h such that the phase rotates § 90° each symbol period

- Based on previous slide, we need h = 1/2
- Note: 1-bit of information per symbol period
  - Bit rate = symbol rate

# A More Convenient Model for Analysis



Same as previous model, but we represent data as impulses convolved with a rectangular pulse

Note that h = 1/2 for MSK

### Impact of Sending a Single Data Impulse



To achieve MSK modulation, resulting phase shift must be +/- 90° (i.e., π/4)

#### Include Influence of Transmit Filter



M.H. Perrott

# **Gaussian Minimum Shift Keying**

#### Definition

Minimum shift keying in which the transmit filter is chosen to have a Gaussian shape (in time and frequency) with bandwidth = B Hz

$$p(t) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{t}{\sigma}\right)^2}$$

where : 
$$\sigma = \frac{.833T_d}{(BT_d)2\pi}$$

#### Key parameters

- Modulation index: as previously discussed
  - h = 1/2

**BT<sub>d</sub> product:** ratio of transmit filter bandwidth to data rate

• For GSM phones:  $BT_d = 0.3$ 

# Modeling The Impact of VCO Phase Modulation

#### **Recall unmodulated VCO model** Phase Noise $S_{out}(f)$ Phase/Frequency **Spurious** Overall modulation Signal Noise phase noise $S_{\Phi_{mod}}(f)$ $\Phi_{tn}(t)$ $|_{\cap}$ $\Phi_{\rm out}$ out(t) $\Phi_{\mathsf{mod}}(\mathfrak{l})$ $2\cos(2\pi f_0 t + \Phi_{out}(t))$

Relationship between sine wave output and instantaneous phase

$$out(t) = 2\cos(2\pi f_o t + \Phi_{out}(t))$$

#### Impact of modulation

Same as examined with VCO/PLL modeling, but now we consider  $\Phi_{out}(t)$  as sum of *modulation* and noise components

$$\Phi_{out}(t) = \Phi_{mod}(t) + \Phi_{tn}(t)$$

#### **Relationship Between Sine Wave Output and its Phase**

#### Key relationship

$$out(t) = \cos(2\pi f_o t + \Phi_{mod}(t) + \Phi_{tn}(t))$$

Using a familiar trigonometric identity

$$out(t) = \cos(2\pi f_o t + \Phi_{mod}(t)) \cos(\Phi_{tn}(t))$$
$$-\sin(2\pi f_o t + \Phi_{mod}(t)) \sin(\Phi_{tn}(t))$$

• Approximation given  $|\Phi_{tn}(t)| << 1$ 

$$out(t) \approx \cos(2\pi f_o t + \Phi_{mod}(t))$$
$$-\sin(2\pi f_o t + \Phi_{mod}(t))\Phi_{tn}(t)$$

Approximation from previous slide

 $out(t) \approx \cos(2\pi f_o t + \Phi_{mod}(t))$ 

 $-\sin(2\pi f_o t + \Phi_{mod}(t))\Phi_{tn}(t)$ 

 Autocorrelation (assume modulation signal independent of noise)

$$R\{out(t)\} = R\{\cos(2\pi f_o t + \Phi_{mod}(t))\}$$

 $+R\{\sin(2\pi f_ot + \Phi_{mod}(t))\}R\{\Phi_{tn}(t)\}$ 

 Output spectral density (Fourier transform of autocorrelation)

$$S_{out}(f) = S_{out_m}(f) + S_{out_m}(f) * S_{\Phi_{tn}}(f)$$

Where \* represents convolution and

 $S_{out_m}(f) = S\{\cos(2\pi f_o t + \Phi_{mod}(t))\}, \ S_{\Phi_{tn}}(f) = S\{\Phi_{tn}(t)\}$ 

## Impact of Phase Modulation on the Output Spectrum



Spectrum of output is distorted compared to S<sub>pmod</sub>(f)
Spurs converted to phase noise

# Leveraging a Fractional-N Synth for Phase Modulation



- Provides a practical means of achieving accurate phase modulation
- Primarily digital structure
  - Analog components consist of charge pump, loop filter, and VCO

# Linearized Model of Fractional-N Modulator



Increases impact of Sigma-Delta and Charge Pump noise

Tradeoff between data rate and noise performance

## Improving the Data Rate/Noise Tradeoff



- Compensation filter allows data rate to exceed PLL bandwidth
  - Allows higher data rates

**M.H. Perrott** Improves SNR and out-of-band emission performance

#### The Issue of Mismatch



Mismatch between compensation filter and PLL forms parasitic pole/zero pair

Causes intersymbol interference (ISI)

# Example of ISI Due to Mismatch



Frequency modulation is fairly insensitive to mismatch

Phase modulation is much more sensitive

#### Is There An Alternate Means of Increasing Data Rate?

## **Classical Fractional-N Synthesizer Architecture**



- Use an accumulator to perform dithering operation
  - Fractional input value fed into accumulator
  - Carry out bit of accumulator fed into divider

Integer-N Synthesizer Signals with  $F_{out} = 4.25F_{ref}$ 



Constant divide value of N = 4 leads to frequency error

Error pulse widths increase as phase error accumulates

Fractional-N Synthesizer Signals with F<sub>out</sub> = 4.25F<sub>ref</sub>



Dithering allows average divide value of N = 4.25

- Reset phase error by periodically "swallowing" a VCO cycle
  - Achieved by dividing by 5 every 4 reference cycles

# Key Observations for Classical Fractional-N Dithering



- The instantaneous phase error always remains less than one VCO cycle
- We can directly relate the phase error to the residue of the accumulator that is providing the dithering

# **Accumulator Operation**



- Carry out bit is asserted when accumulator residue reaches or surpasses its full scale value
- Accumulator residue corresponds to instantaneous phase error
  - Increments by the fractional value input into the accumulator

#### The Issue of Spurious Tones



- PFD error waveform is periodic
  - Creates spurious tones in synthesizer output at lower frequencies than the reference
  - Ruins noise performance of the synthesizer

## The Phase Interpolation Technique



- Leverage the fact that the phase error due to fractional technique is predicted by the instantaneous residue of the accumulator
  - Cancel out phase error based on accumulator residue

## The Problem With Phase Interpolation



- Gain matching between PFD error and scaled D/A output must be extremely precise
  - Any mismatch will lead to spurious tones at PLL output

Matching issue prevented this technique from catching on

#### **Examine Classical Fractional-N Signals**



#### Goal: eliminate the fractional spurs

# Method 1: Vertical Compensation



"Fill in" pulses so that they are constant area

Fractional spurs are eliminated!

# Method 2: Horizontal Compensation



M.H. Perrott

# Implementation of Horizontal Cancellation

We begin with the basic fractional-N structure



## Add a Second PFD with Delayed Divider Signal



#### **Scale Error Pulses According to Accumulator Residue**



# A Closer Look at Adding the Scaled Error Pulses



Goal – keep area constant for each pulse

It's easier to see this from a slightly different viewpoint

## Alternate Viewpoint

The sum of scaled pulses can now be viewed as horizontal cancellation


## Implementation of Pulse Scaling Operation

Direct output of a differential current DAC into two charge pumps



Issue: practical non-idealities kill performance

## **Primary Non-idealities of Concern**



Proposed approach: dramatically reduce impact of these non-idealities using mixed-signal processing techniques

M.H. Perrott

## Eliminate Impact of DAC Current Element Mismatch

Apply standard DAC noise shaping techniques to shape mismatch noise to high frequencies

See Baird and Fiez, TCAS II, Dec 1995



## Eliminate Impact of Timing Mismatch

- Swap paths between divider outputs in a pseudorandom fashion
  - Need to also swap ε[k] and 1-ε[k] sequence



M.H. Perrott

## Improve Horizontal Cancellation Performance

Sampling circuit accumulates error pulses before passing their information to the loop filter

A common analog trick used for decades



M.H. Perrott

Application: A 1 MHz Bandwidth Fractional-N Frequency Synthesizer Implementation

### **Design Goals**

- Output frequency: 3.6 GHz
  - Allows dual-band output (1.8 GHz and 900 MHz)
- Reference frequency: 50 MHz
  - Allows low cost crystal reference
- Bandwidth: 1 MHz
  - Allows fast settling time and ~1 Mbit/s modulation rate
- Noise: < -150 dBc/Hz at 20 MHz offset (3.6 GHz carrier)</p>
  - Phase noise at the 20 MHz frequency offset is very challenging for GSM and DCS transmitters
    - GSM: -162 dBc/Hz at 20 MHz offset (900 MHz carrier)
    - DCS: -151 dBc/Hz at 20 MHz offset (1.8 GHz carrier)

Simultaneous achievement of the above bandwidth and noise targets is very challenging

## **Evaluate Noise Performance with 1 MHz PLL BW**

#### G(f) parameters

- <sup>-</sup> 1 MHz BW, Type II, 2<sup>nd</sup> order rolloff, extra pole at 2.5 MHz
- Required PLL noise parameters (with a few dB of margin)
  - Output-referred charge pump noise: -105 dBc/Hz
  - VCO noise: -155 dBc/Hz at 20 MHz offset (3.6 GHz carrier)

Dynamic Parameters	paris. pole 2.5e6		On	Noise Parameters					
fo 1e6 Hz	paris. Q		On	ref. freq	50e6	 Hz			
order C1 © 2 C 3	paris. pole	Hz	On	out freq.	3.6e9	— Hz			
shape  • Butter  • Bessel • Cheby1  • Cheby2  • Elliptical	paris.Q	_	On	Detector	-105	 dBc/Hz	On		
	paris.pole	Hz	On	vco	-155	dBc/Hz	0n		
	paris. pole	Hz	On	freq. off:	set 20e6	Hz			
type 01 02	paris.zero	Hz	Un	S-D_O_	© 2 _ On		On		
tz/to 1/3	paris. zero	Hz	Un	030	04 05		_		
Resulting Open Loop Parameters Resulting Plots and Jitter									
K: 2.885e+012 alter: On				C Pole/Zero Diagram C Transfer Function					
fp: 2.807e+006 Hz alter: On		Appl	y I	10-2 100-6 -170 -60					
fz: 1.111e+005 Hz alte	er: On		10000 -170 -60						
	Qp: alter: On			rms jitter: 2.197 ps					
Qp: alte	er: On		rms j	iller. <b>  2.1</b> 97					
Qp: alte	er: On On	Written I	rms j by Micł	nter. <b>2.19</b> 7 nael Perrott (f	nttp://www-mtl.mit.edu	/~perrott)			

## **Calculated Phase Noise for Classical Fractional-N**



## Calculated Phase Noise for 7-bit PFD/DAC Synth



## Simulation of PFD/DAC Synthesizer using CppSim



Phase noise plots to follow: 40e6 time steps in 11 min

## **2<sup>nd</sup> Order** $\Sigma$ - $\Delta$ **Fractional-N Performance**



**Replace accumulator with second order**  $\Sigma - \Delta$  **modulator** 

Set residue into PFD/DAC equal to zero

## Calculate PLL Noise for $2^{nd}$ Order $\Sigma - \Delta$ Synthesizer



М.п. геноц

#### Simulated Phase Noise of 2<sup>nd</sup> Order *Σ*–*Δ* Synthesizer



M.H. Perrott

## 7-bit PFD/DAC Synthesizer Performance



- Use accumulator for dithering
- Enable cancellation by connecting accumulator residue to PFD/DAC

## Calculate PLL Noise for 7-bit PFD/DAC Synthesizer



M

#### Simulated PLL Phase Noise of 7-bit PFD/DAC



PLL Design Assistant accurately models simulated noise! M.H. Perrott

53

# A 1 MHz BW Fractional-N Frequency Synthesizer IC



Implements proposed 7-bit **PFD/DAC** structure

0.18u CMOS

**Circuit details** published in **VLSI 2005** 

Funded by MARCO C2S2

Op-amp and Sample Network

> Fabricated by **National** Semiconductor

## **Measured Noise Suppression**



- Comparison of 7-bit PFD/DAC synthesizer with 2<sup>nd</sup> order ΣΔ Synthesizer
- Low freq noise ~2dB worse because of phase swapping
- 29dB quantization noise suppression measured at 10MHz !

M.H. Perrott

# A Highly Digital Implementation of a GMSK Transmitter



- A fractional-N frequency synthesizer provides highly accurate phase/frequency modulation capability
  - Multiple carrier frequencies easily achieved with digital frequency division
  - N-bit PFD/DAC extends achievable data rate for a given noise performance (at higher frequency offsets)

## GMSK Eye Diagrams at 271 kbit/s (~900 MHz Carrier)



#### **Close agreement between simulated and measured results!**

M.H. Perrott

## GMSK Spectra Plots at 271 kbit/s (~900 MHz Carrier)



## GMSK Eye Diagrams at 500 kbit/s (~900 MHz Carrier)



## GMSK Spectra Plots at 500 kbit/s (~900 MHz Carrier)



## GMSK Eye Diagrams at 1 Mbit/s (~900 MHz Carrier)



## GMSK Spectra Plots at 1 Mbit/s (~900 MHz Carrier)



## **Conclusions**

- Fractional-N frequency synthesizers can achieve dramatic improvement in achieving high PLL bandwidth with excellent noise performance
  - The PFD/DAC approach presented here is only one of many possibilities to achieve this goal
- Design and simulation methodologies are invaluable for achieving better performance
  - Analytical modeling of noise can be quite accurate
    - The PLL Design Assistant can be useful in this area
  - Behavioral simulation can be used to verify analytical models
    - CppSim offers a convenient and fast framework for this

#### Simulation of Frequency Synthesizers

## Impact of Synthesizer Noise



## Impact of Synthesizer Dynamic Behavior



Settling time must be fast to support channel hopping requirements

### What Do We Want From a Simulator?

- Accurate estimation of synthesizer performance
  - Noise spectral density
  - Dynamic behavior
- Fast computation to allow use in IC design flow
- Simple to use
  - C++, Verilog, Matlab

# Problems with Current Simulators

## **Problem 1: Classical Simulators are Slow**



- High output frequency is High sample rate
- Long time constants → Long time span for transients

Large number of simulation time steps required

## **Problem 2: Classical Simulators Are Inaccurate**



- PFD output is not bandlimited
  - PFD output must be simulated in discrete-time
- Phase error is inaccurately simulated
- Non-periodic dithering of divider complicates matters

## Example: Classical Constant-Time Step Method



- Directly sample the PFD output according to the simulation sample period
  - Simple, fast, readily implemented in Matlab, Verilog, C++
- Issue quantization noise is introduced
  - This noise overwhelms the PLL noise sources we are trying to simulate

## Alternative: Event Driven Simulation



#### Set simulation time samples at PFD edges

Sample rate can be lowered to edge rate!
# Issue: Simulation of Filter Blocks is Complicated



- Filtering computation must deal with non-constant time step
  - Closed-form calculation is tedious
  - Iterative computation is time-consuming
  - Complicates Verilog, Matlab, or C++ implementation

# Is there a better way?

### **Proposed Approach: Use Constant Time Step**



- Straightforward CT to DT transformation of filter blocks
  - Use bilinear transform or impulse invariance methods
- Overall computation framework is fast and simple
  - Simulator can be based on Verilog, Matlab, C++

#### **Problem: Quantization Noise at PFD Output**



- Edge locations of PFD output are quantized
  - Resolution set by time step: T<sub>s</sub>
- Reduction of T<sub>s</sub> leads to long simulation times

#### **Proposed Approach: View as Series of Pulses**



- Area of each pulse set by edge locations
- Key observations:
  - Pulses look like impulses to loop filter
  - Impulses are parameterized by their area and time offset

# **Proposed Method**



- Set e[n] samples according to pulse areas
  - Leads to very accurate results
    - Mathematical analysis given in paper
  - Fast computation



Compute transition values in VCO block

# **Calculation of Transition Values**



#### Model VCO based on its phase

# **Calculation of Transition Values**



#### Determine output transition time according to phase

#### **Calculation of Transition Values**



Use first order interpolation to determine transition value



- Compute transition values in VCO block
- Pass transition information in Divider block



- Compute transition values in VCO block
- Pass transition information in Divider block
- Compute transition values for PFD output



- Compute transition values in VCO block
- Pass transition information in Divider block
- Compute transition values for PFD output
- Compute Filter output



- Compute transition values in VCO block
- Pass transition information in Divider block
- Compute transition values for PFD output
- Compute Filter output

# **Computation of PFD Output**



- Goal: compute transition information in terms of primitive blocks (registers, XOR gates, etc.)
  - Allows straightforward implementation in simulator
  - Accommodates a rich variety of PFD structures

# Implementation of Primitives - Registers



Relevant timing information is contained in the clock signal

- Transfer transition information from the clock to the register output
- Complement output using a sign change

# Implementation of Primitives – Logic Gates



- Relevant timing information contained in the input that causes the output to transition
  - Determine which input causes the transition, then pass its transition value to the output

# Issue: Must Observe Protocol When Adding Noise



- Divider and PFD blocks operate on a strict protocol for their incoming signals
  - Values other than 1 or -1 are interpreted as edges
  - Example: inputting noise at divider input breaks protocol!
- Add noise only at places where signal is "analog"
  - **PFD**, charge pump, and loop filter outputs are fine

M.H. Perrott

# Can we speed the simulation up further?

# Sample Rate Set by Highest Frequency Signal



Time step of simulation typically set by VCO output

Small time steps means long simulation runs



 Divider output often 100 times lower in frequency

Can we sample according to divider output?

# **Divider Output Can Be Computed from VCO Phase**



(Van Halen et al, Circuits and Systems '96)

Key Idea: Model VCO and Divider using Phase

# **Combine VCO and Divider Blocks**



#### Transient simulations run 2 orders of magnitude faster!

# Does it really work?

# The CppSim Simulator

- Blocks are implemented with C/C++ code
  - High computation speed
  - Complex block descriptions
- Users enter designs in graphical form using Cadence or Sue2 schematic capture
  - System analysis and transistor level analysis are possible in the same CAD framework
- Resulting signals are viewed in Matlab or CppSimView
  - Powerful post-processing and viewing capability

#### Simulation package freely downloadable at http://www.cppsim.com

# The Sue2 and CppSimView Environment



M.H. Perrott

97

#### **Experimental Prototype to Verify Approach**



# Simulation Results - Dynamic Behavior



Simulation time: 260 thousand time steps in 5 seconds on a 650 MHz Pentium III Laptop (custom C++ simulator)

# **Noise Sources Included in Simulation**



Dominant noise sources in synthesizer

- **Quantization noise of**  $\Sigma \Delta$  (produced by  $\Sigma \Delta$  block)
- Charge pump noise (calculated from Hspice)
- VCO noise (input-referred calculated from measurement)

#### Measured Synthesizer Noise Performance



#### Simulated Synthesizer Noise Performance



Simulated results compare quite well to measured!

Simulation time: 5 million time steps in 80 seconds

#### Conclusion

- Phase locked loop circuits can be quickly and accurately simulated
  - Accuracy achieved with area conservation principle
  - Fast computation by combining VCO and Divider blocks
- A variety of simulation frameworks can be used
  - C++, Matlab, Verilog
  - Circuit primitives are supported

Noise and dynamic performance of fractional-N frequency synthesizers can be investigated at system level